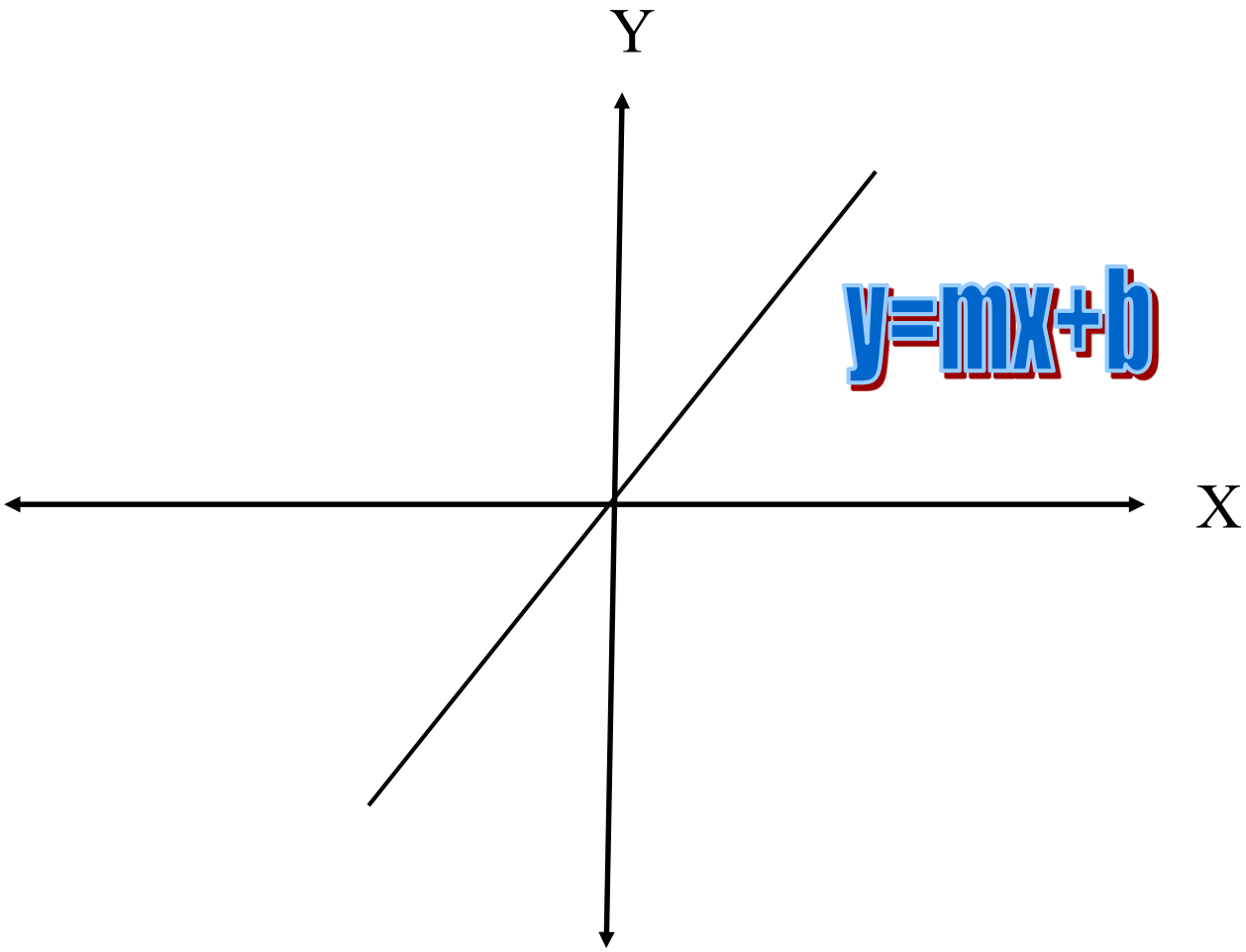
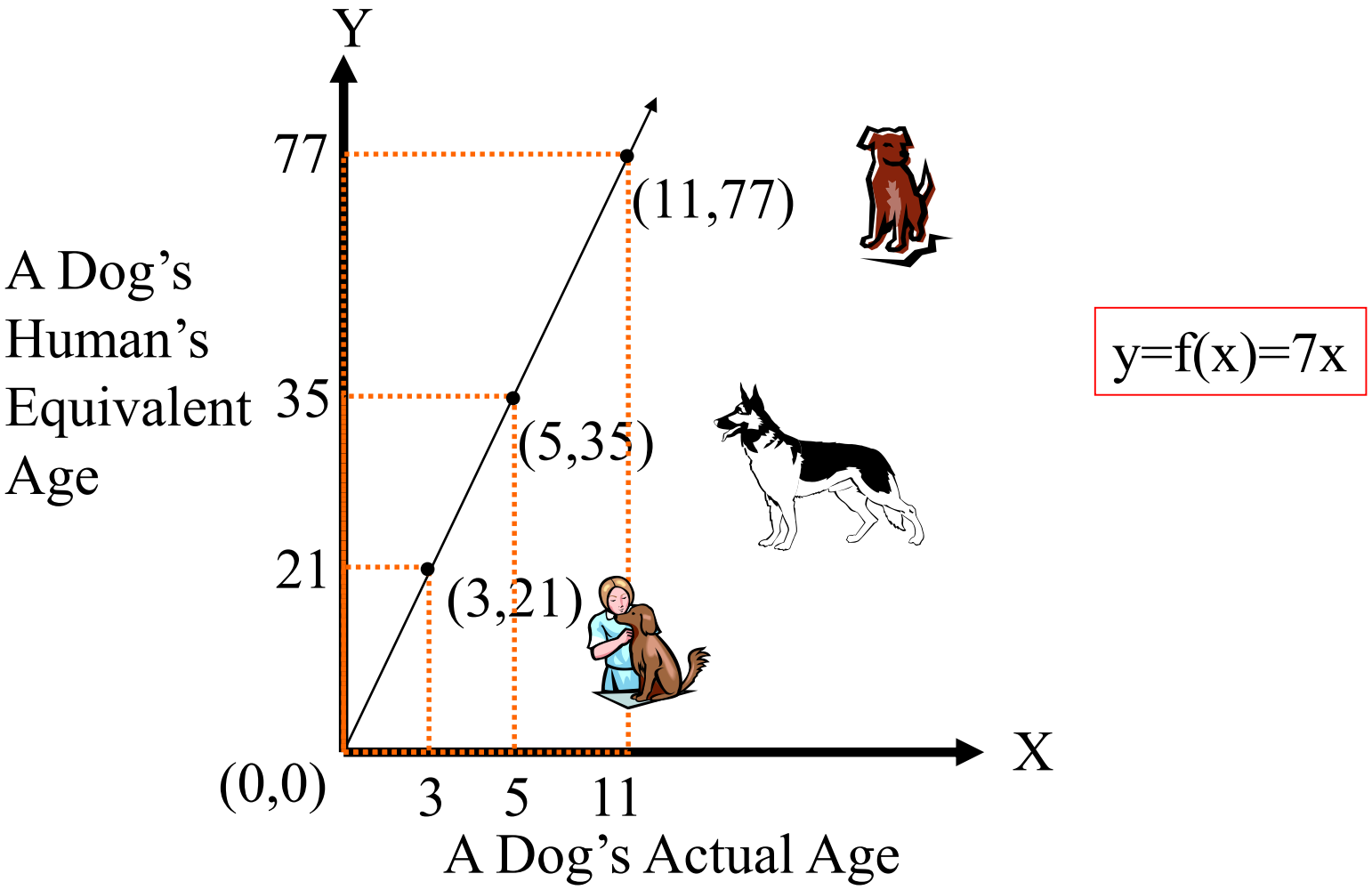
GRAPHS & LINEAR EQUATIONS



Example of a Linear Function

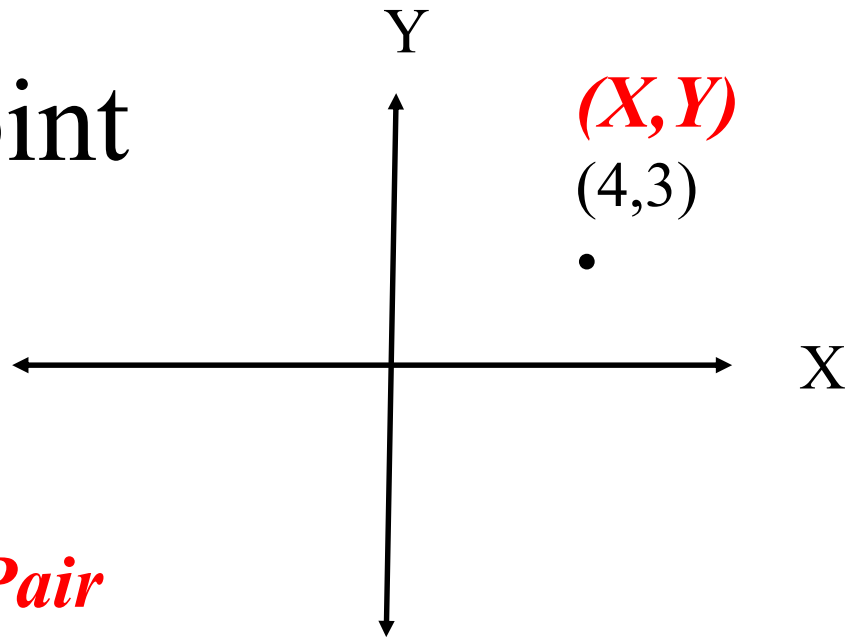


$y=f(x)=7x$

Major Elements of Graphing Lines

- **Graphing Ordered Pairs**
- **Graphing Equations**
- **Linear Equations**
- **Slope & Equations**
- **Finding Equations of Lines**
- **Fitting Equations to Lines**
- **Parallel & Perpendicular Lines**

A Point

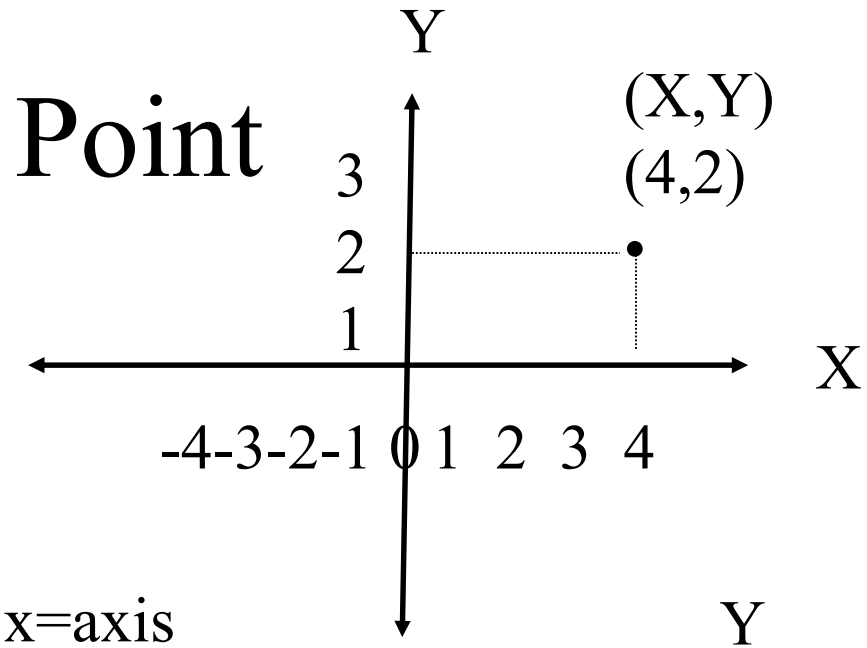


(X,Y) is called an ***Ordered Pair***

The X value or X Coordinate is the location of a point in the X direction

The Y value or Y Coordinate is the location of a point in the Y direction

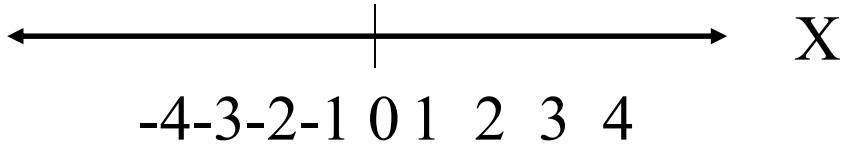
How to Graph a Point



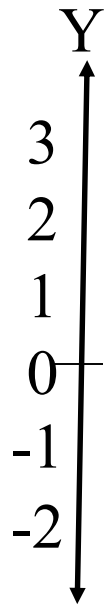
X is the distance along the x=axis

Y is the distance along the y=axis

HINT: Think of the x-axis as the Number Line

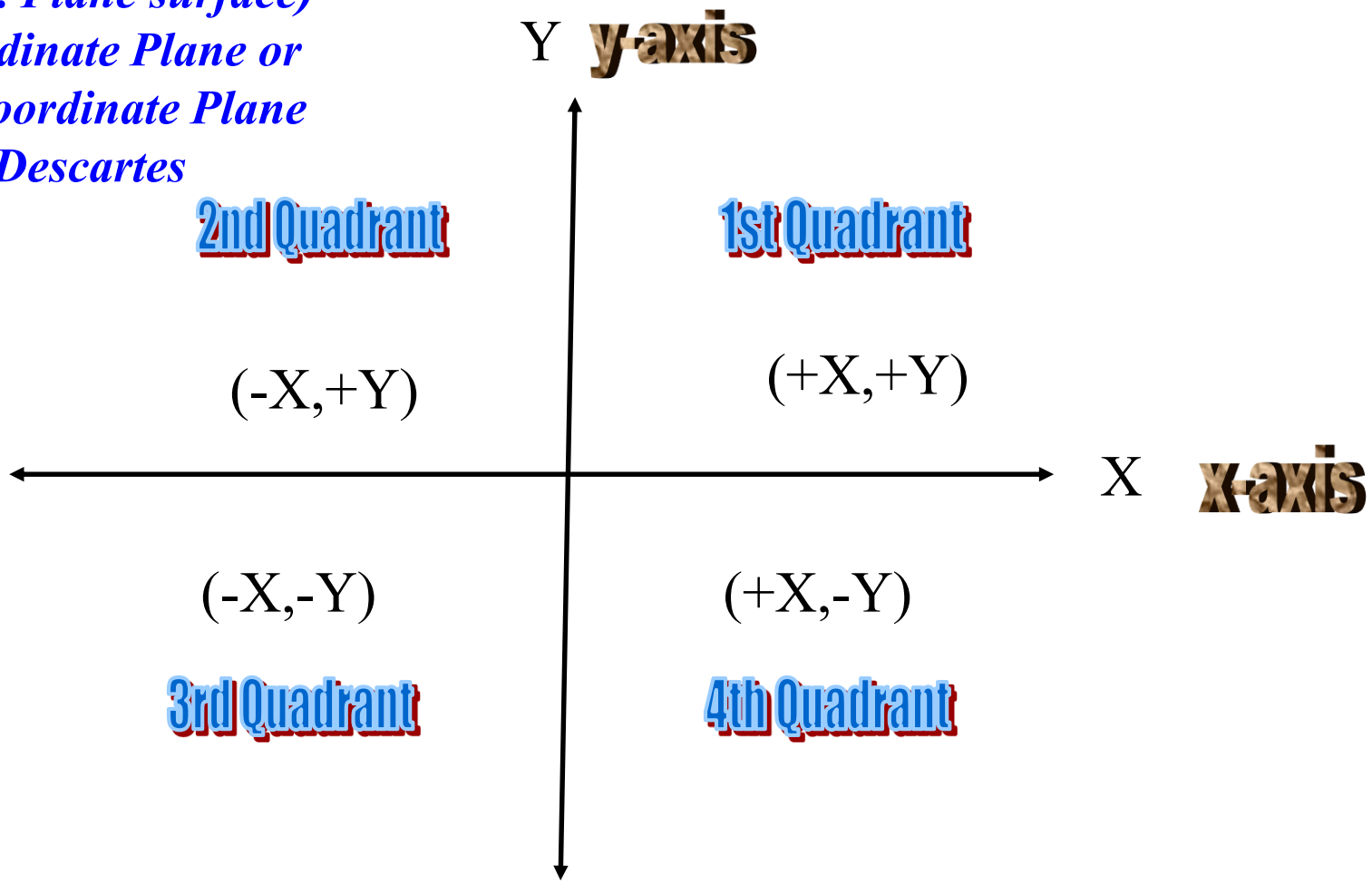


HINT: Think of the y-axis a vertical Number Line



Important Vocabulary for Graphs

The Graph itself is called the x-y plane (ie. Plane surface) or The Coordinate Plane or Cartesian Coordinate Plane after Renee Descartes



Quadrants start with Positive X & Y and go Counter Clockwise

Graphing Linear Equations
(Find 3 Domain & Range Points)

First Degree Equations are Lines (y=mx+b) and you calculate 3 (X,Y) values

Make sure the points line up on a x-y graph and connect the dots.

RECALL X-Domain & Y-Range

Graphing Lines is just like finding the Range of 3 Domain Points:
(Substitute each Domain value into the equation)

$y = 2x - 7$ when the ***Domain is {-2, 0, 2}***

f(-2) = 2•(-2) -7 = -4 -7 = -11 ***(-2,-11)***
f(0) = 2•(0) -7 = 0 -7 = -7 ***(0,-7)***
f(2) = 2•(2) -7 = 4 -7 = -3 ***(2,-3)***

Answer: RANGE: {-11, -7, -3}

Practice Finding 3 Points Given a Linear Equation

Find any 3 (X,Y) points for the following equations:

$y=5x$

$y=4x-5$

$y=3x+1$

(Hint: Try x=0)

Sample Solutions

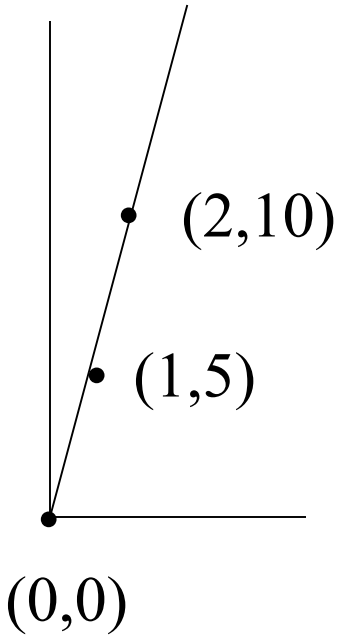
x	y = 5x
0	0
1	5
2	10

x	y = 4x-5

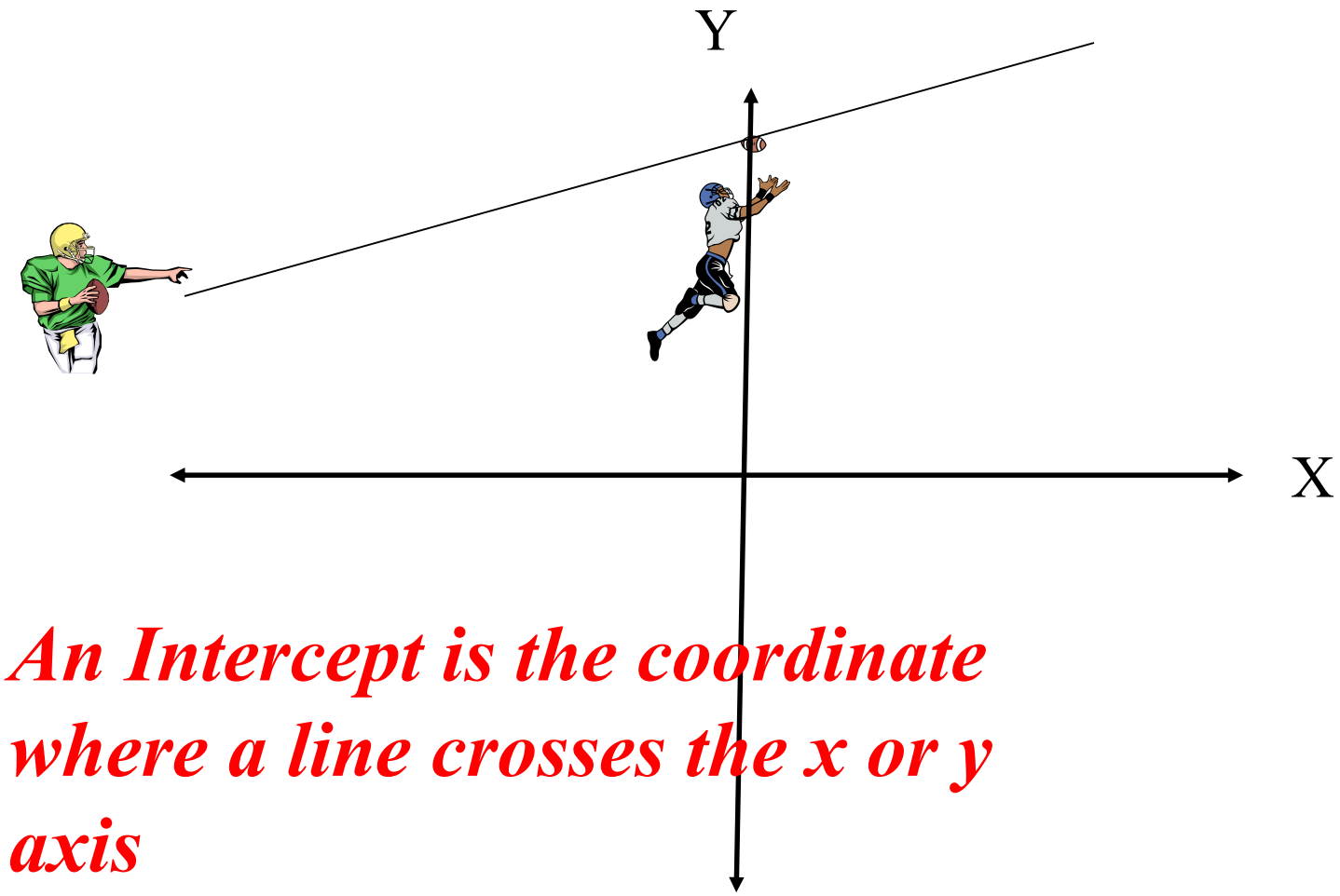
x	y = 3x+1

Now Graph the 3 Points

x	y = 5x
0	0
1	5
2	10



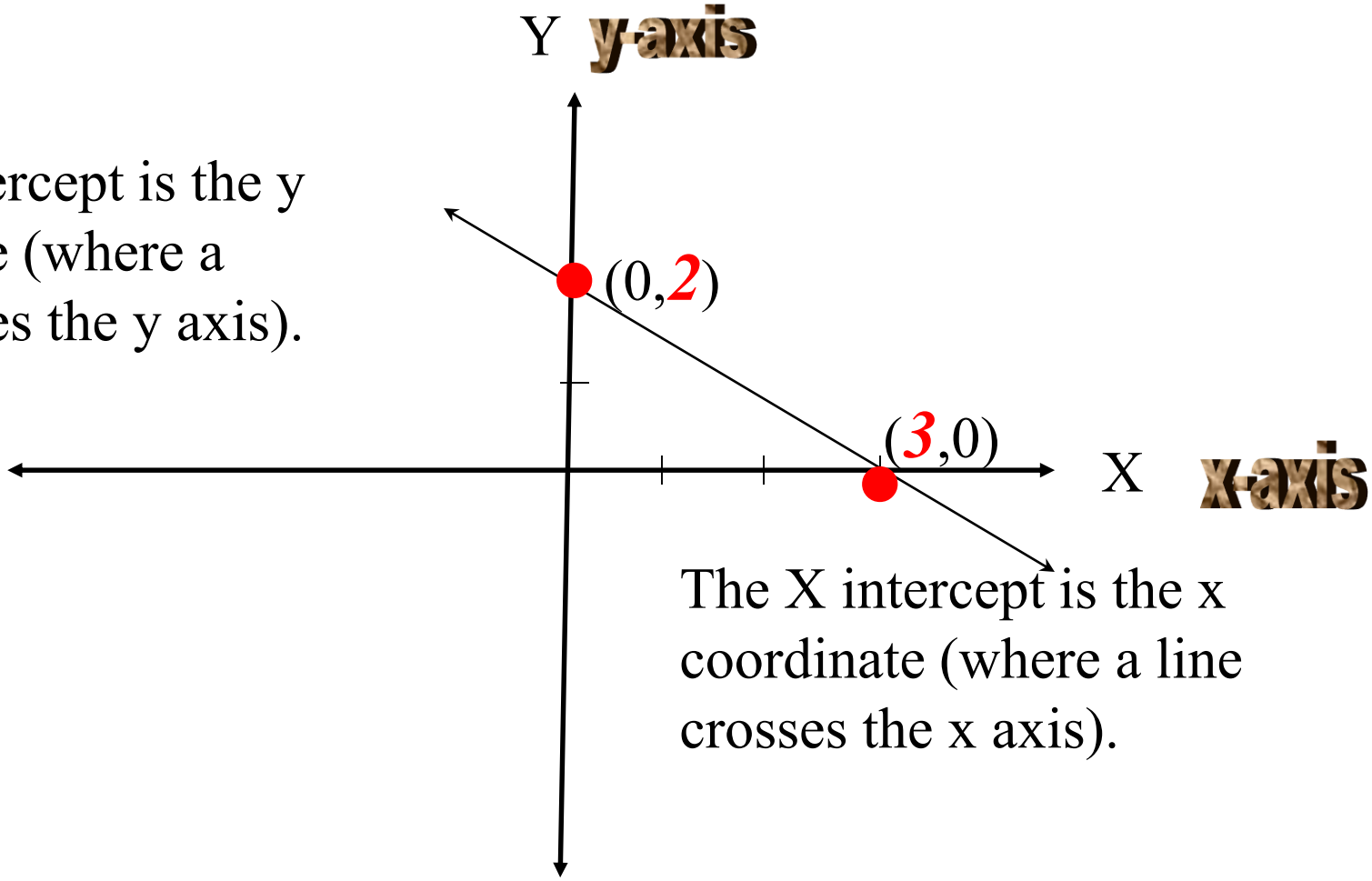
What is Intercept in Math?



An Intercept is the coordinate where a line crosses the x or y axis

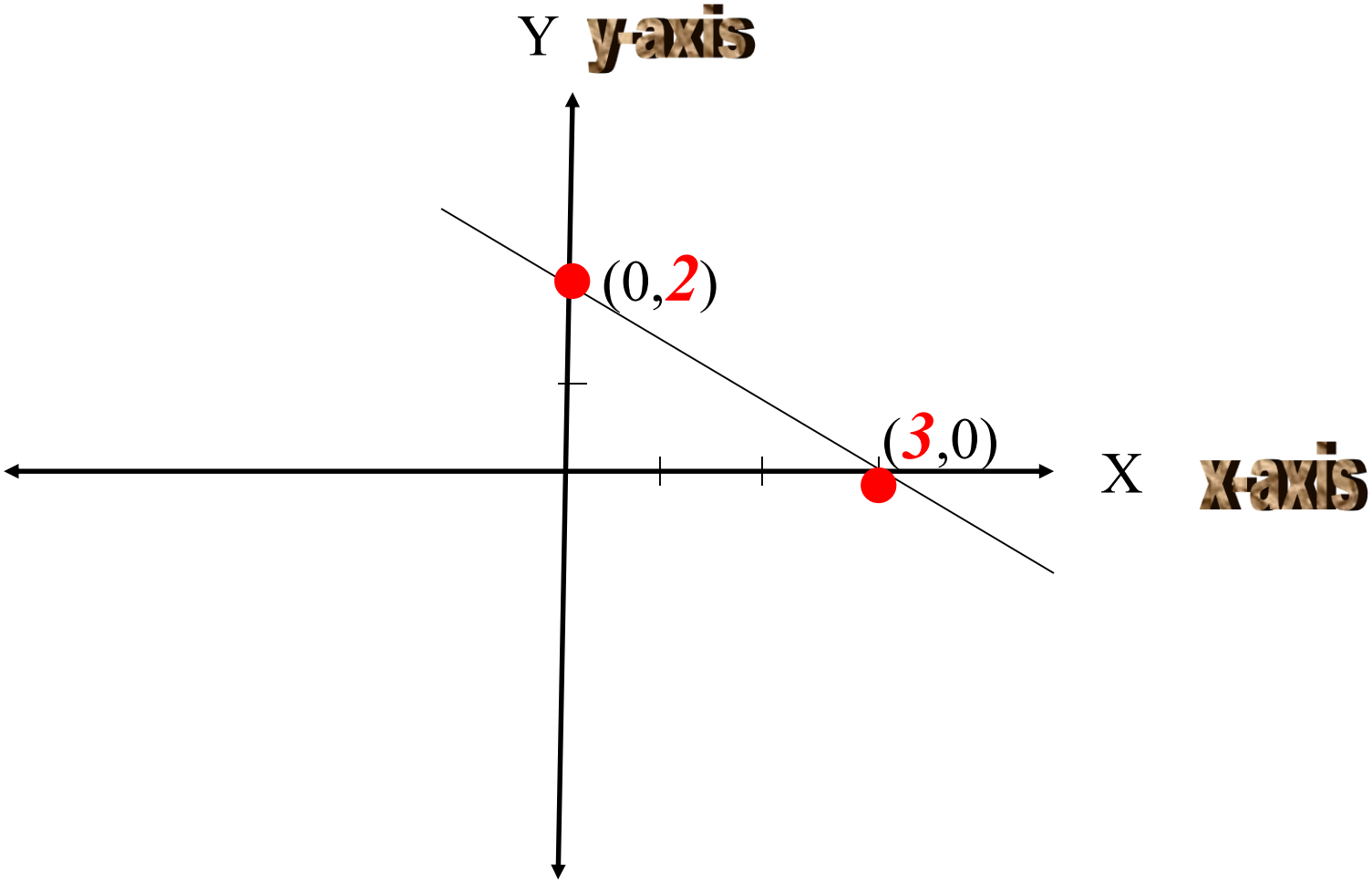
Using X&Y Intercepts to Graph a Line

The Y intercept is the y coordinate (where a line crosses the y axis).

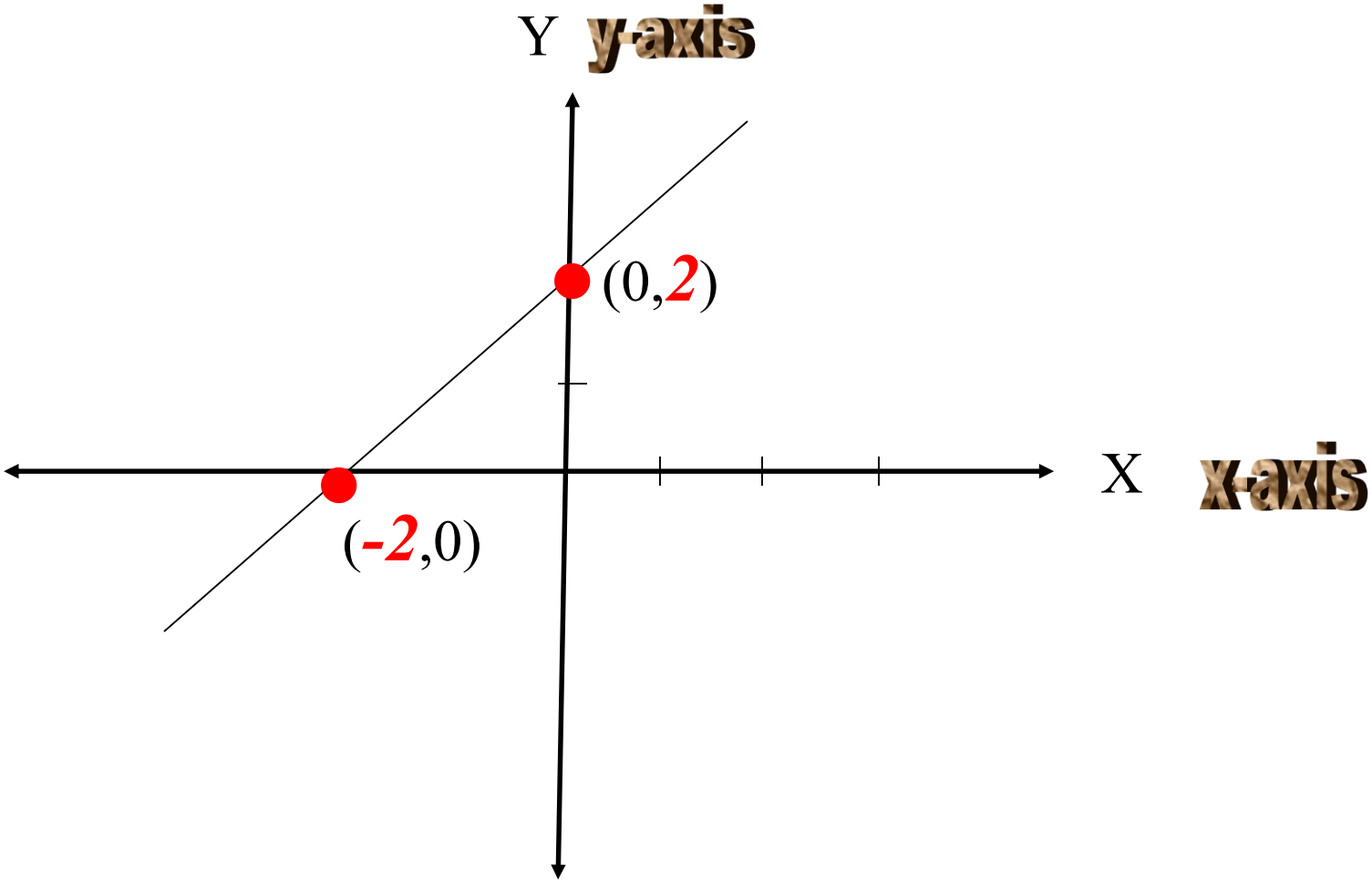


The X intercept is the x coordinate (where a line crosses the x axis).

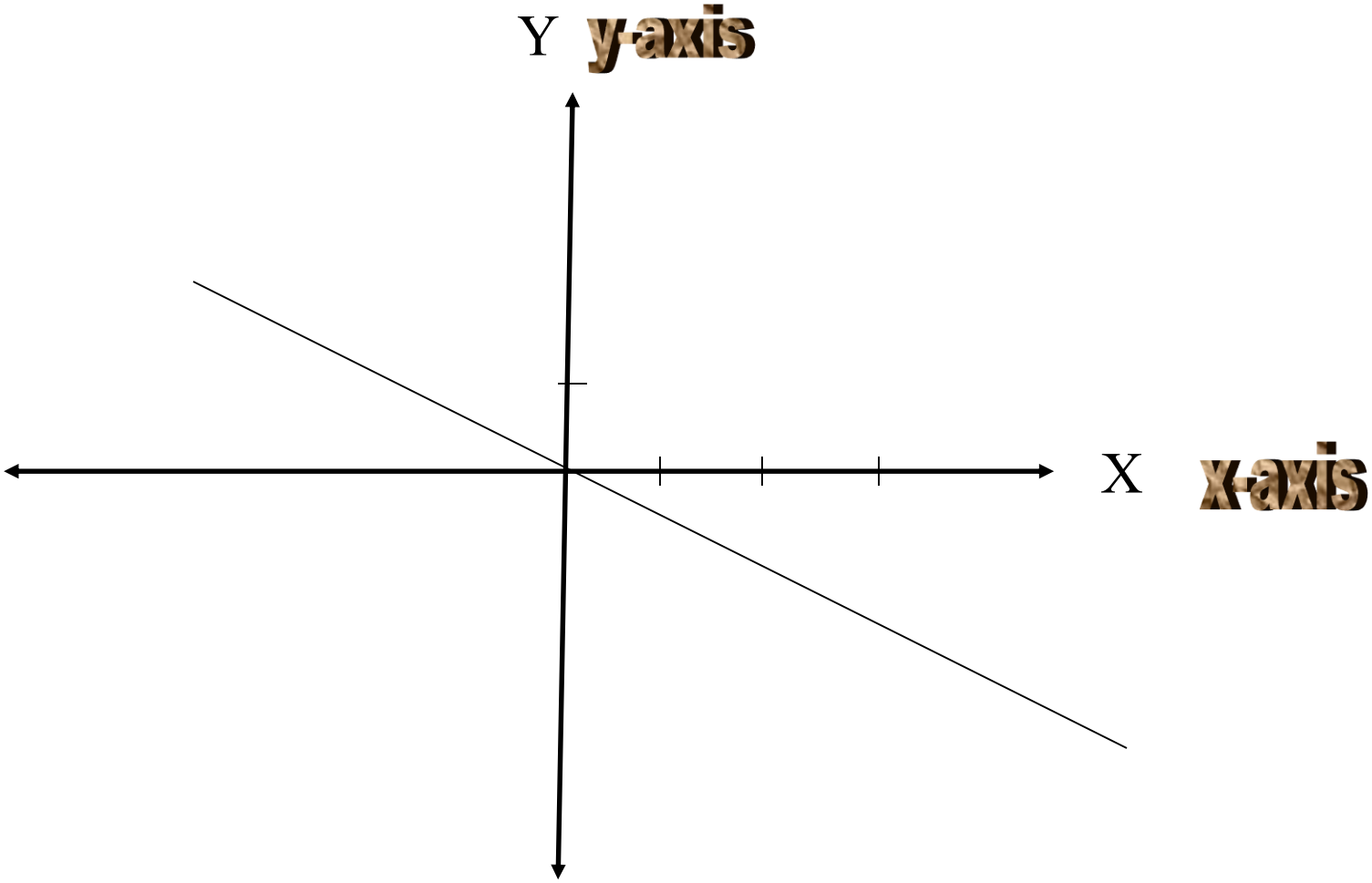
Name the X&Y Intercepts



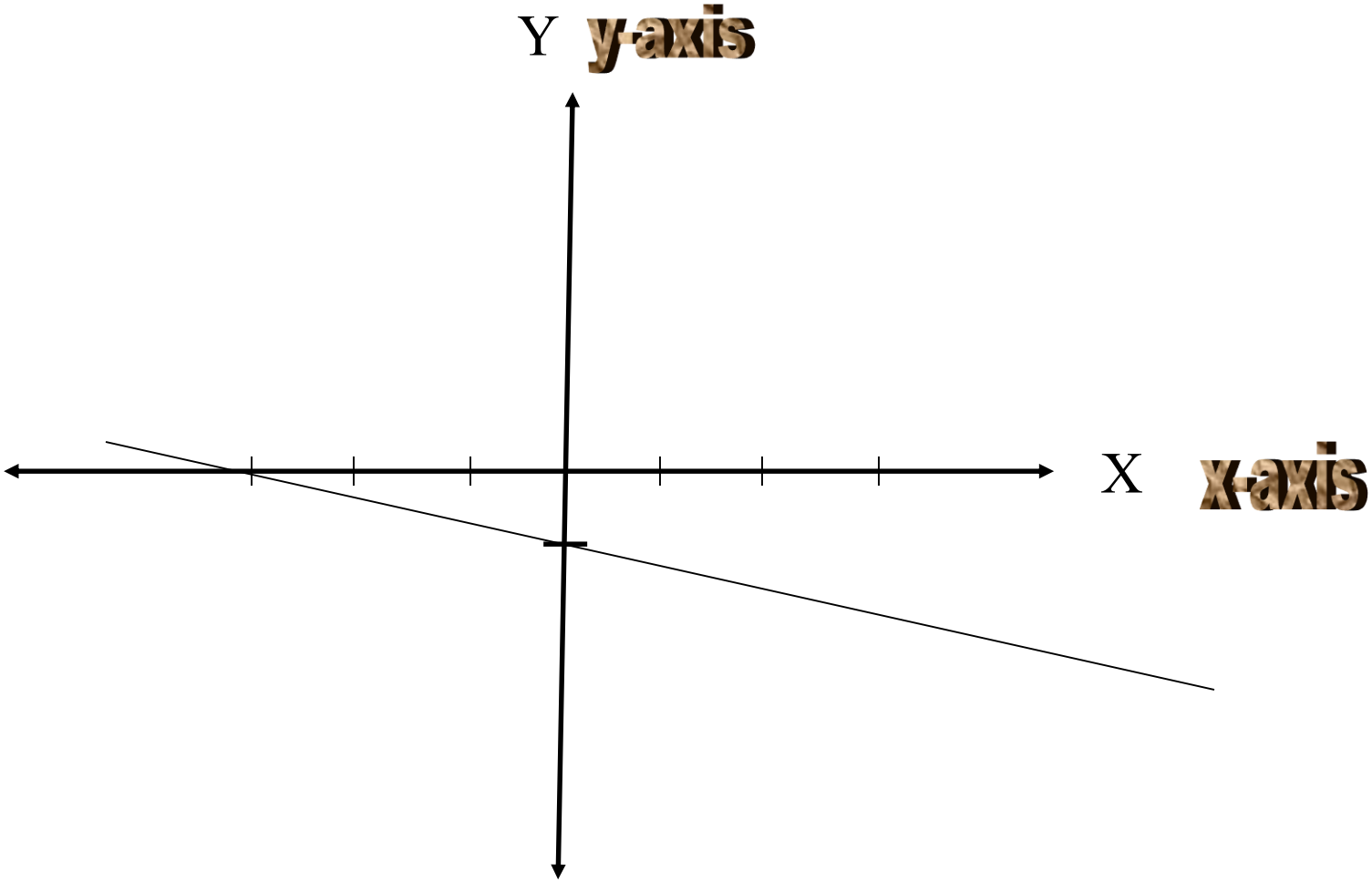
Name the X&Y Intercepts



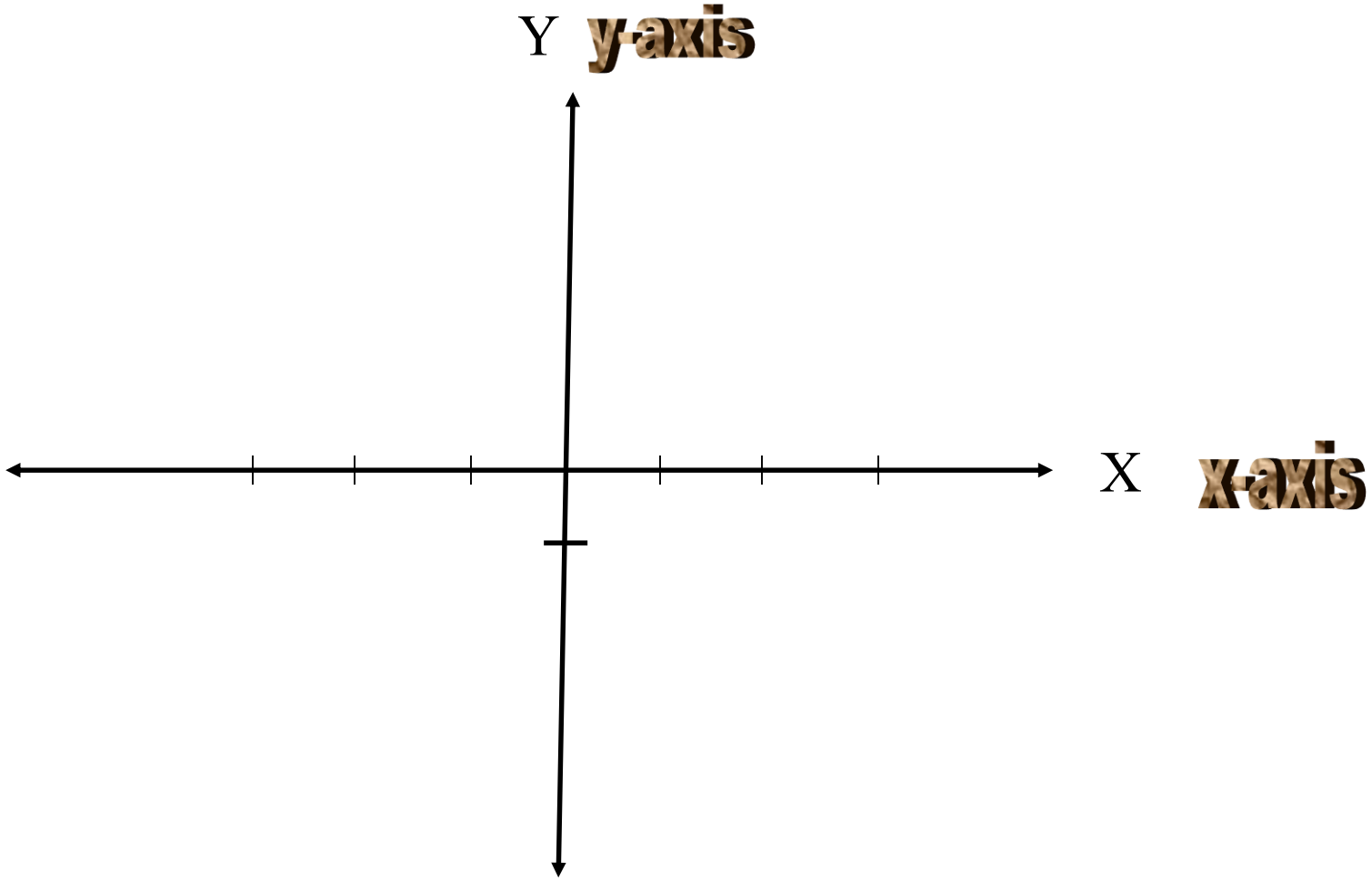
Name the X&Y Intercepts



Name the X&Y Intercepts



What is the value of x at the y intercept?
What is the value of Y at the x-intercept?

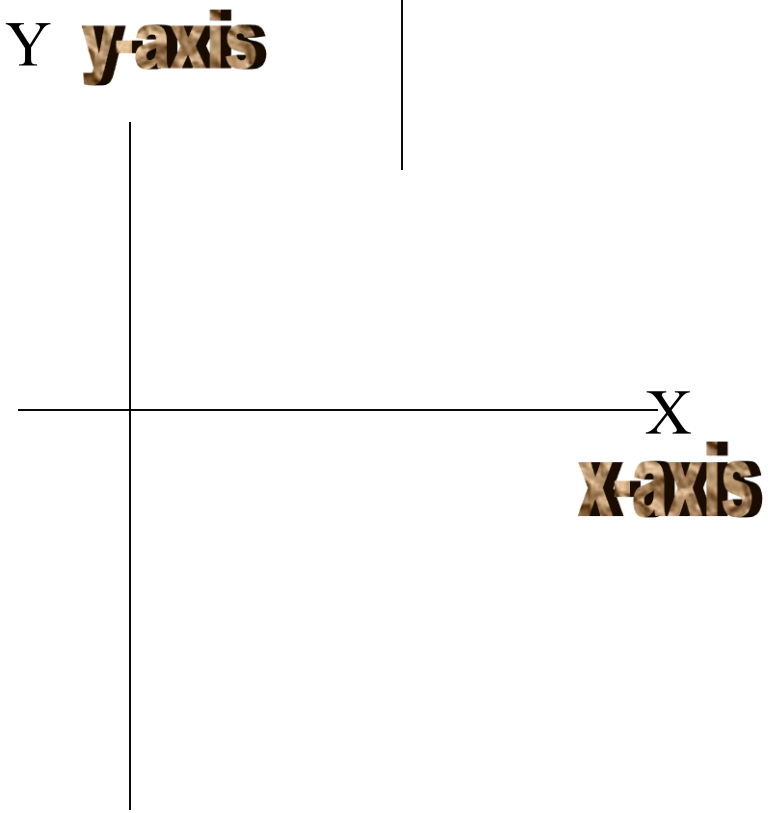


Graph y = 2x - 6 using x&y intercepts

Graph Linear Eq.

X	Y	= 2x - 6

1st *Make x-y table*



Graph y = 2x - 6 using x&y intercepts

Graph Linear Eq.

X	Y	= 2x - 6
0	-6	

Y y-axis

X x-axis

(0,-6)

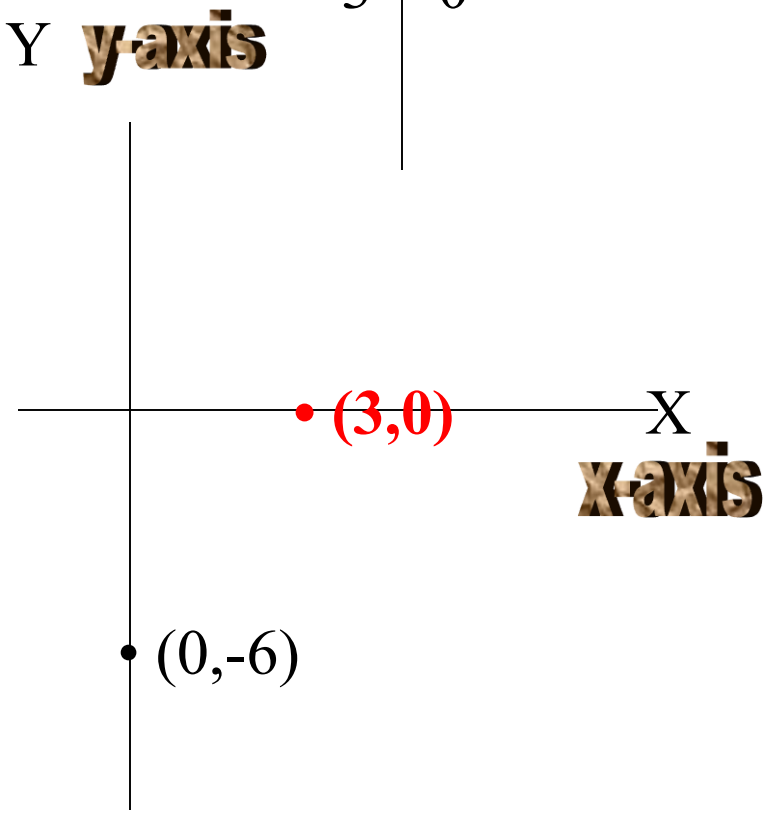
1st *Make x-y table*

2nd *Set x = 0 and solve for y*

Graph y = 2x - 6 using x&y intercepts

X	Y
0	-6
3	0

= 2x - 6



Graph Linear Eq.

1st ***Make x-y table***

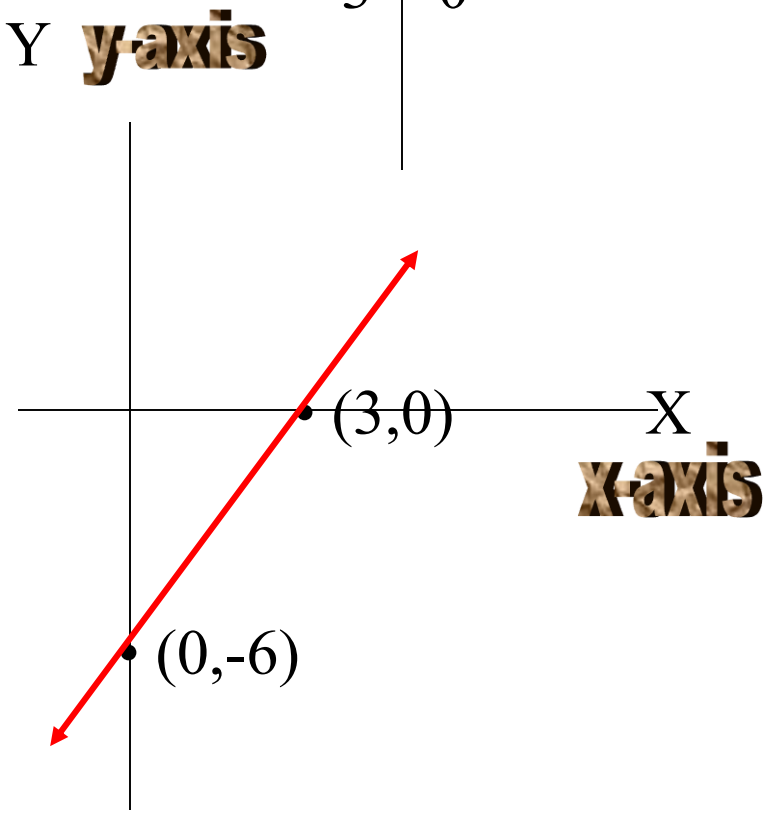
2nd ***Set x = 0 and solve for y***

3rd ***Set y = 0 and solve for x***

Graph y = 2x - 6 using x&y intercepts

X	Y
0	-6
3	0

= 2x - 6



Graph Linear Eq.

1st ***Make x-y table***

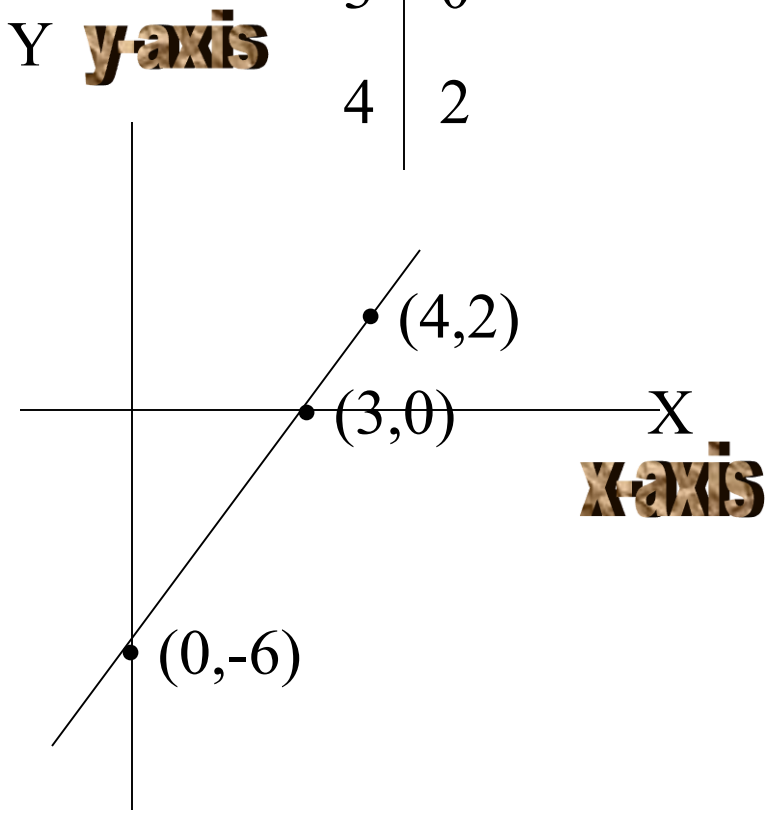
2nd ***Set x = 0 and solve for y***

3rd ***Set y = 0 and solve for x***

4th ***Plot these 2 points and draw line***

Graph y = 2x - 6 using x&y intercepts

X	Y	= 2x - 6
0	-6	
3	0	
4	2	



Graph Linear Eq.

1st *Make x-y table*

2nd *Set x = 0 and solve for y*

3rd *Set y = 0 and solve for x*

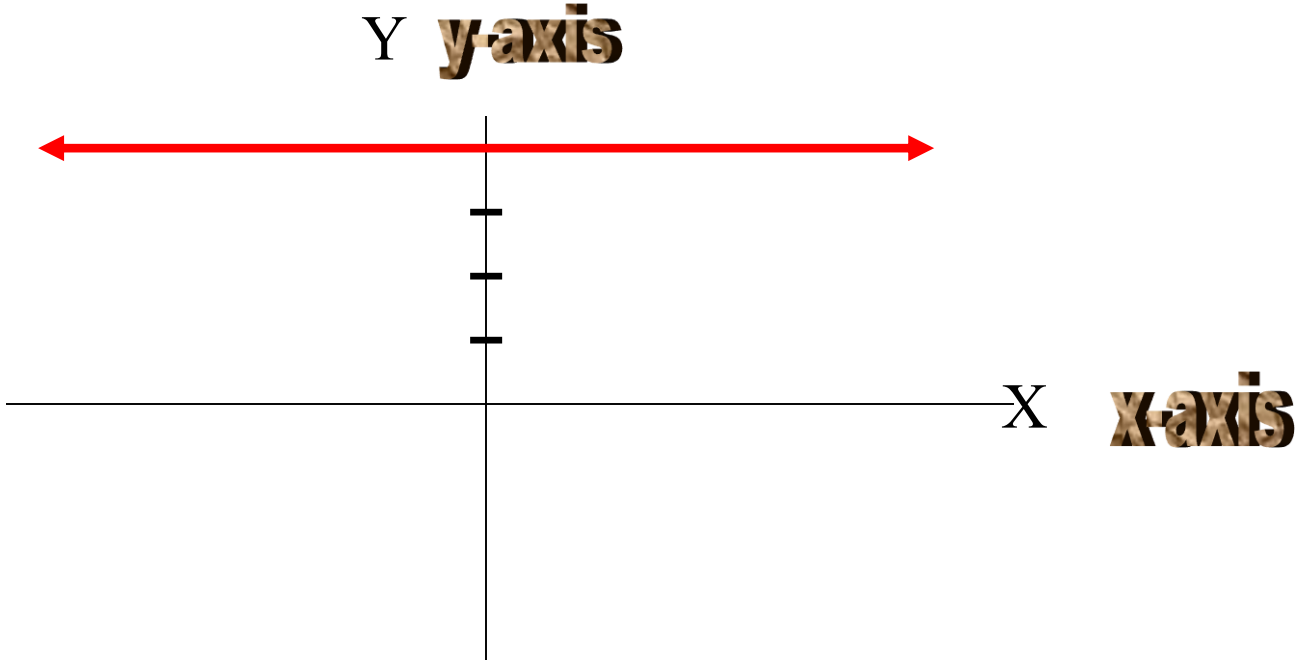
4th *Plot these 2 points and draw line*

5th *Use 3rd point to check*

Graphing Horizontal & Vertical Lines

This line has a y value of 4 for any x-value. It's equation is

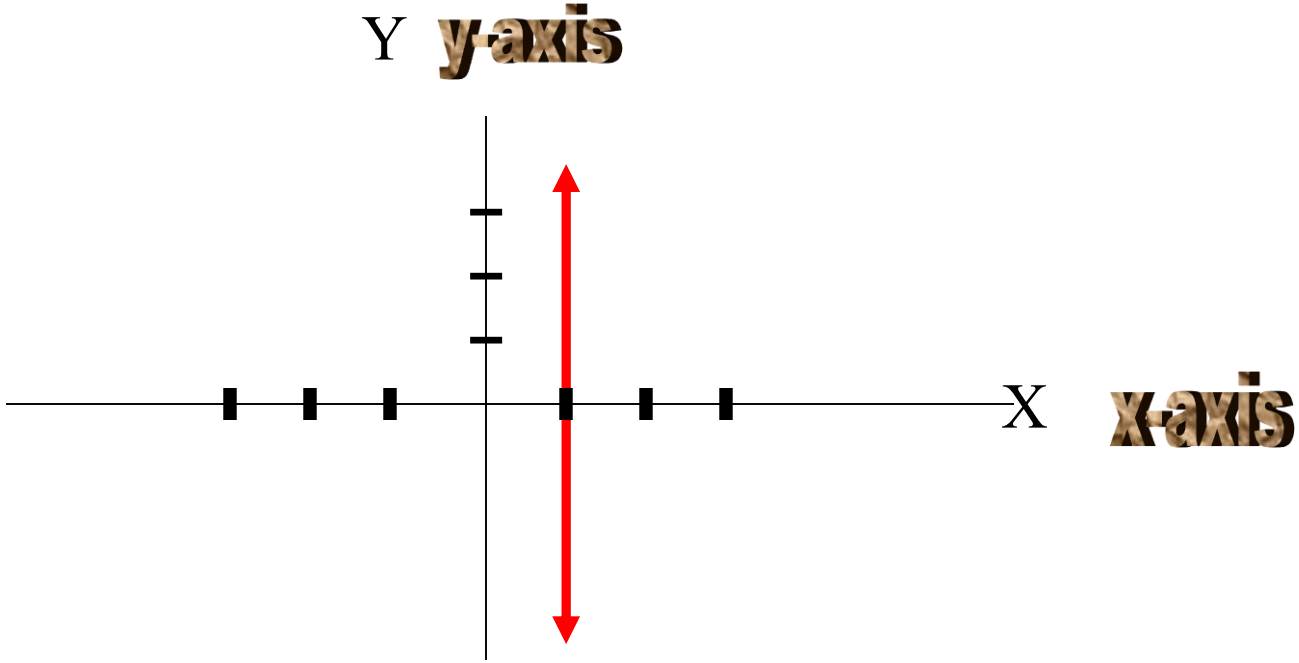
y = 4 (meaning y always equals 4)



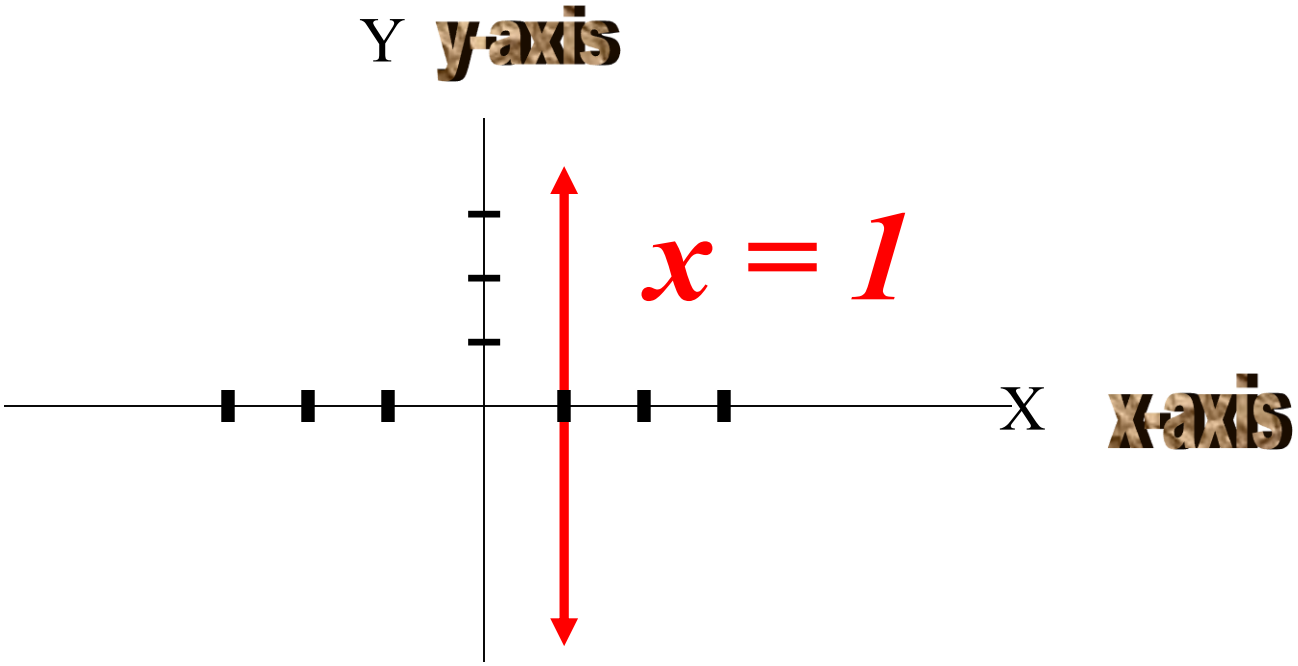
Graphing Horizontal & Vertical Lines

This line has a x value of 1 for any y-value. It's equation is

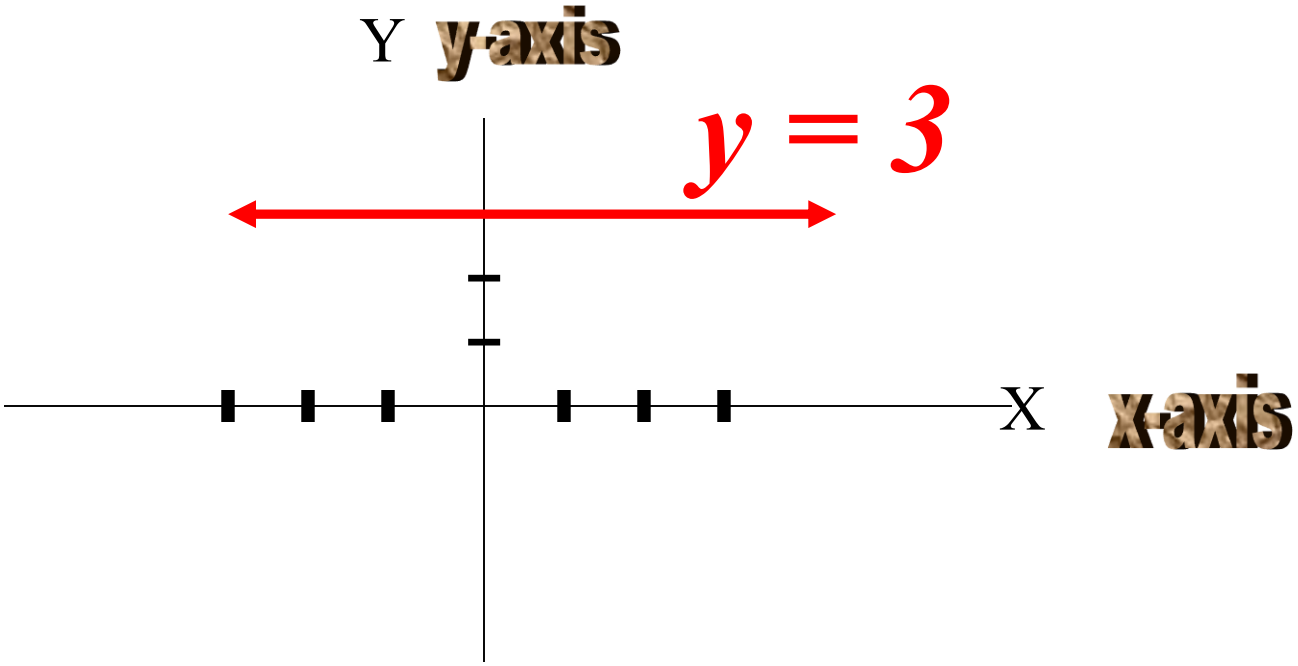
x = 1 (meaning x always equals 1)



The Equation of a Vertical Line is X=Constant



The Equation of a Horizontal Line is Y=Constant



Graph the following lines

$Y = -4$

$Y = 2$

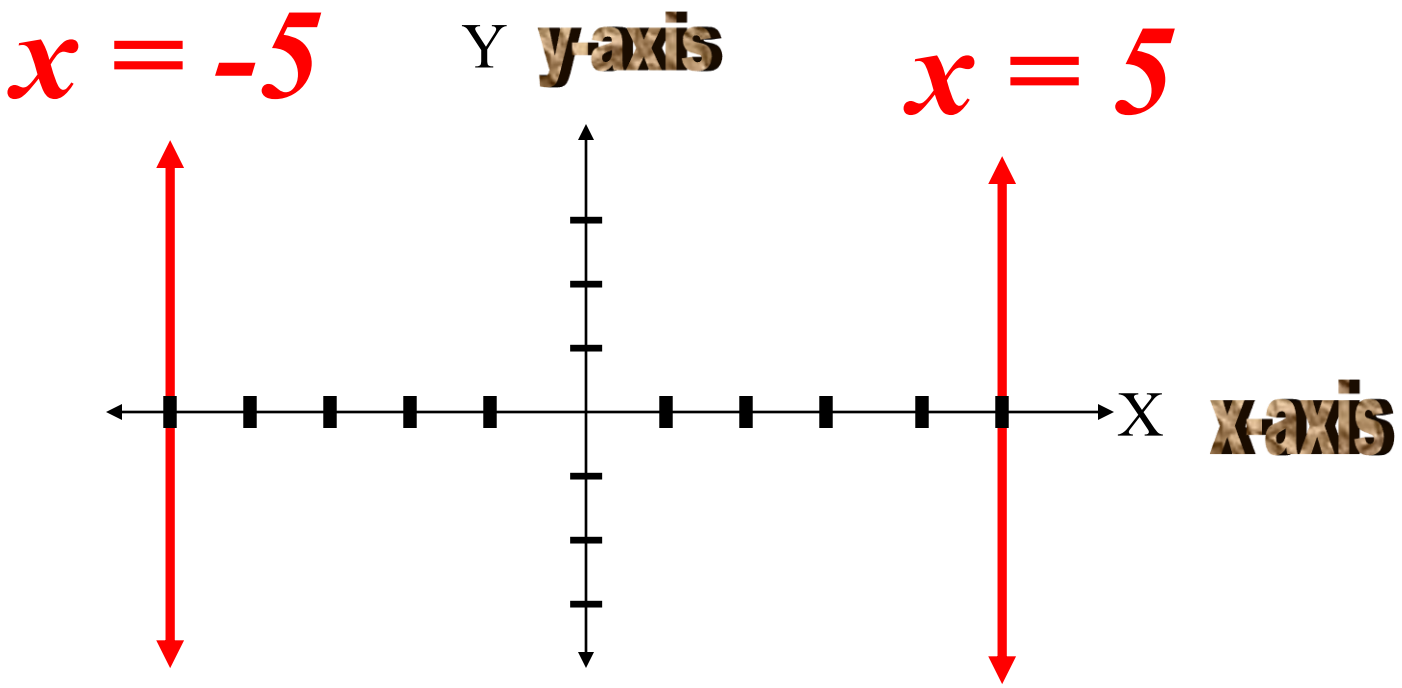
$X = 5$

$X = -5$

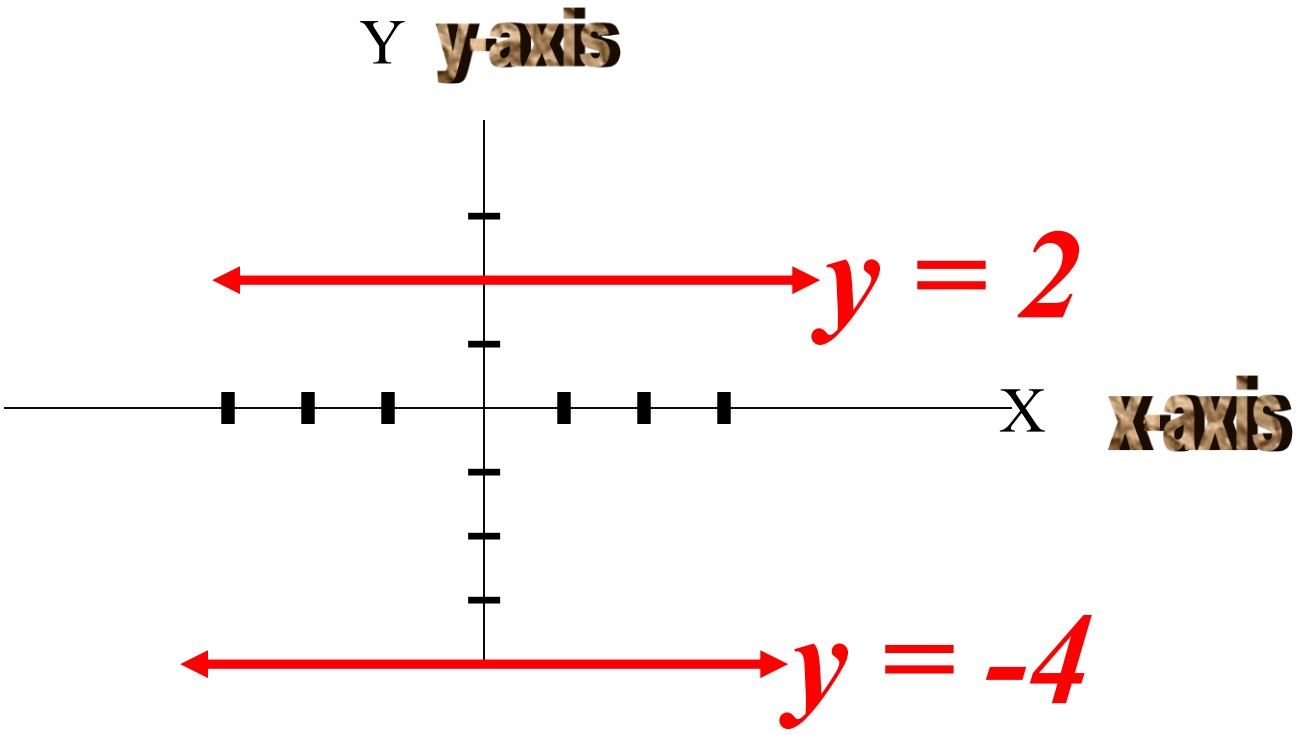
$X = 0$

$Y = 0$

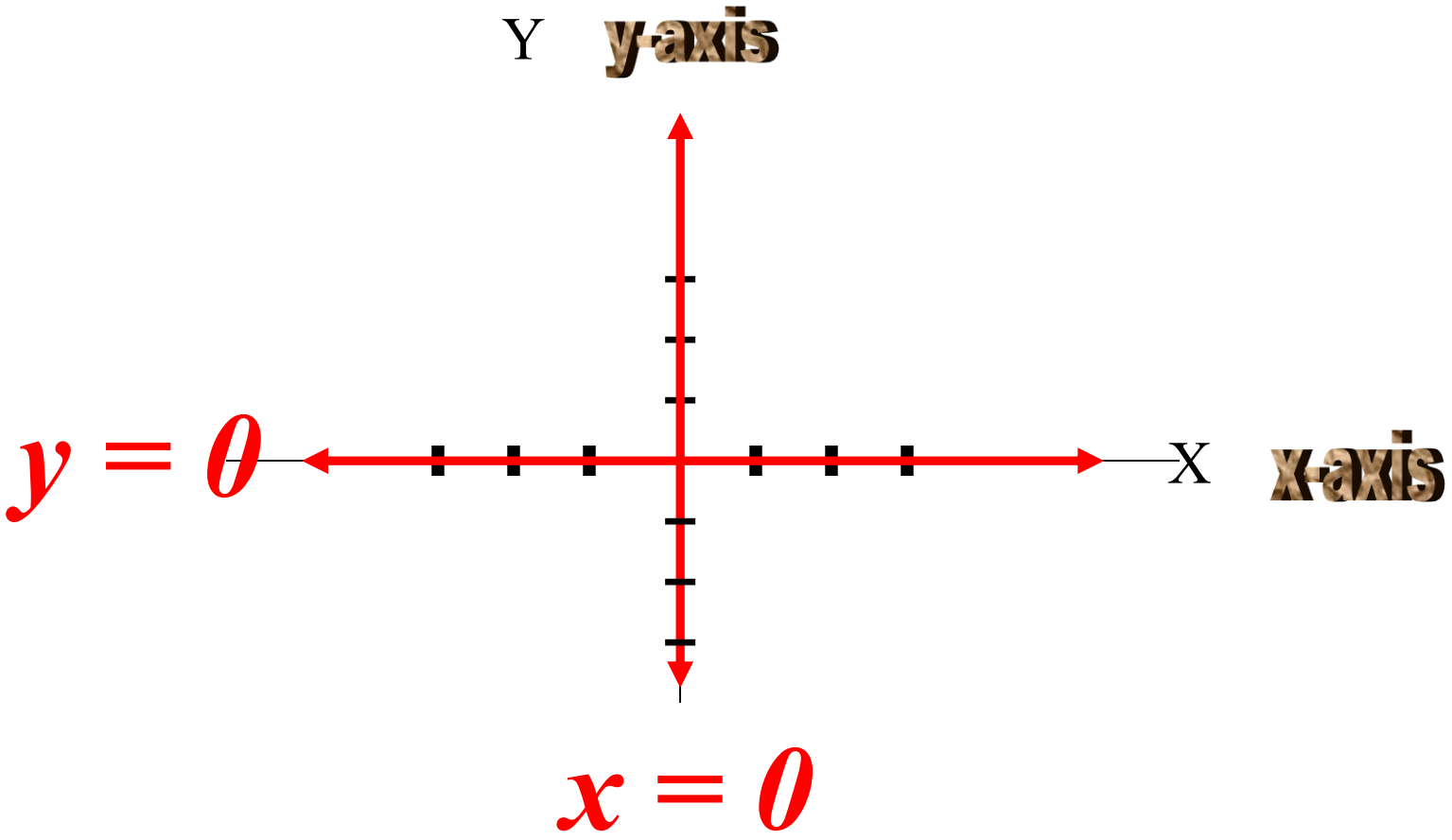
Answers



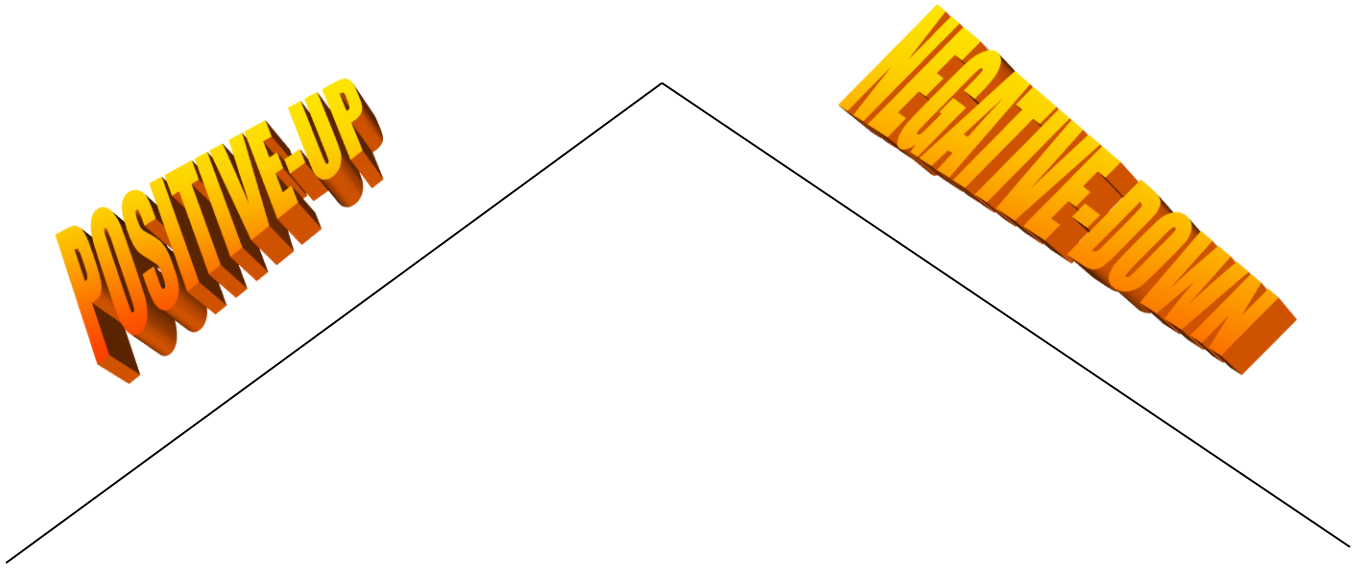
Answers



Answers

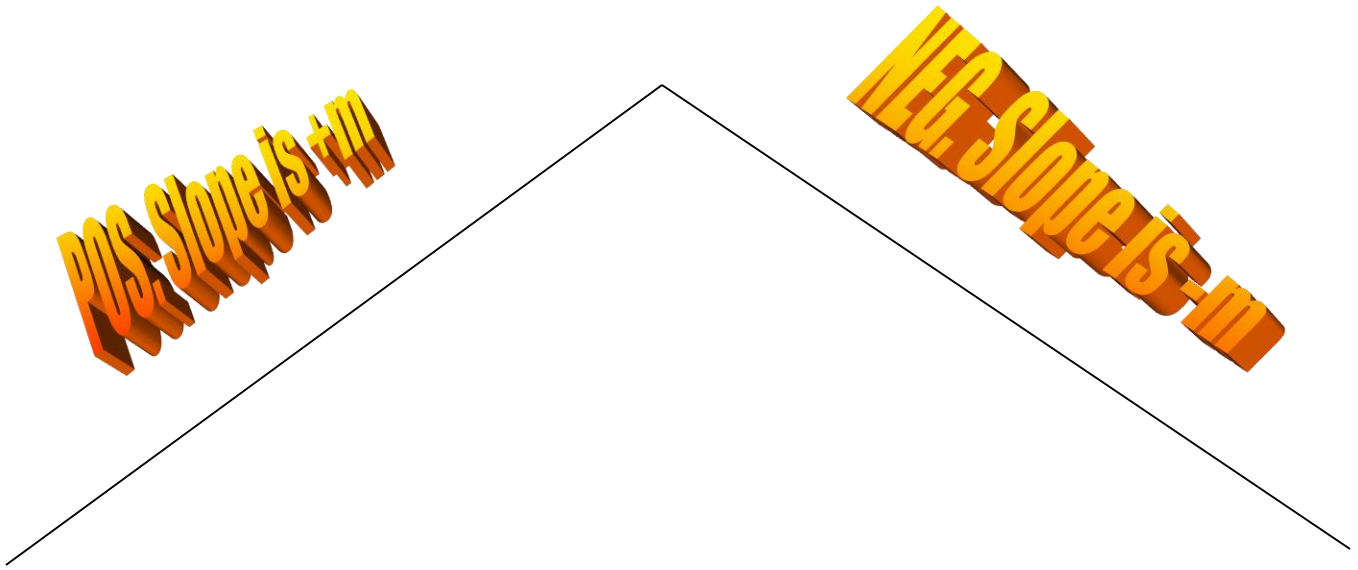


$$\text{SLOPE} = \frac{RISE}{RUN}$$



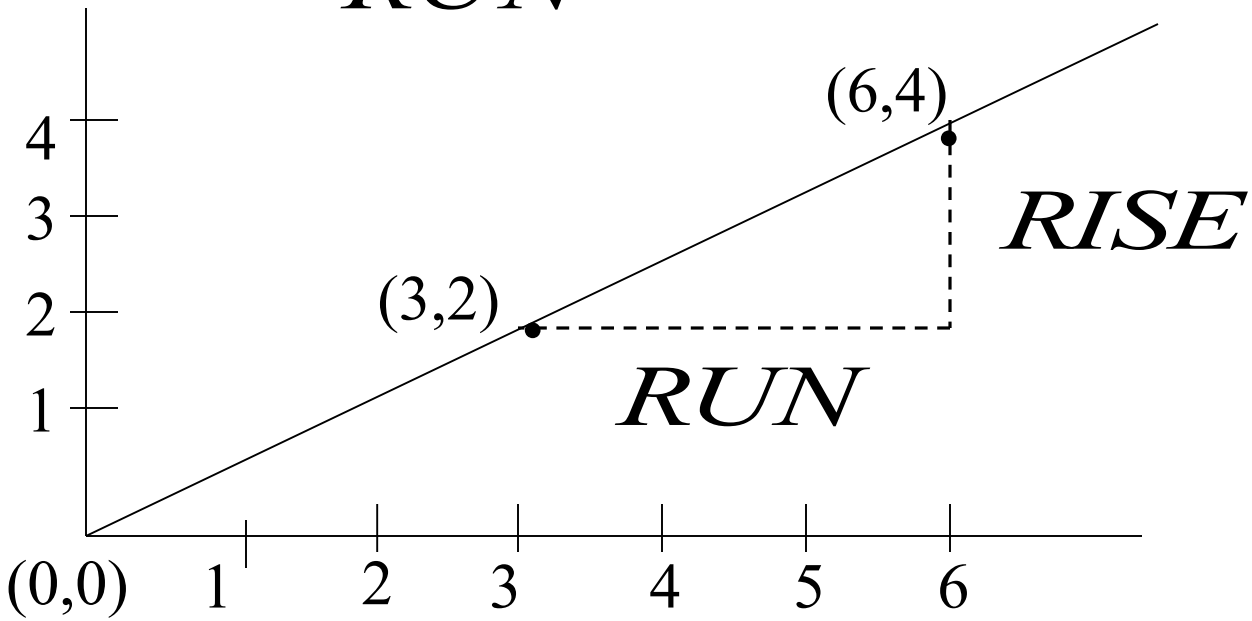
Slope is a measure of STEEPNESS

The Symbol for SLOPE = m



Think of m for Mountain

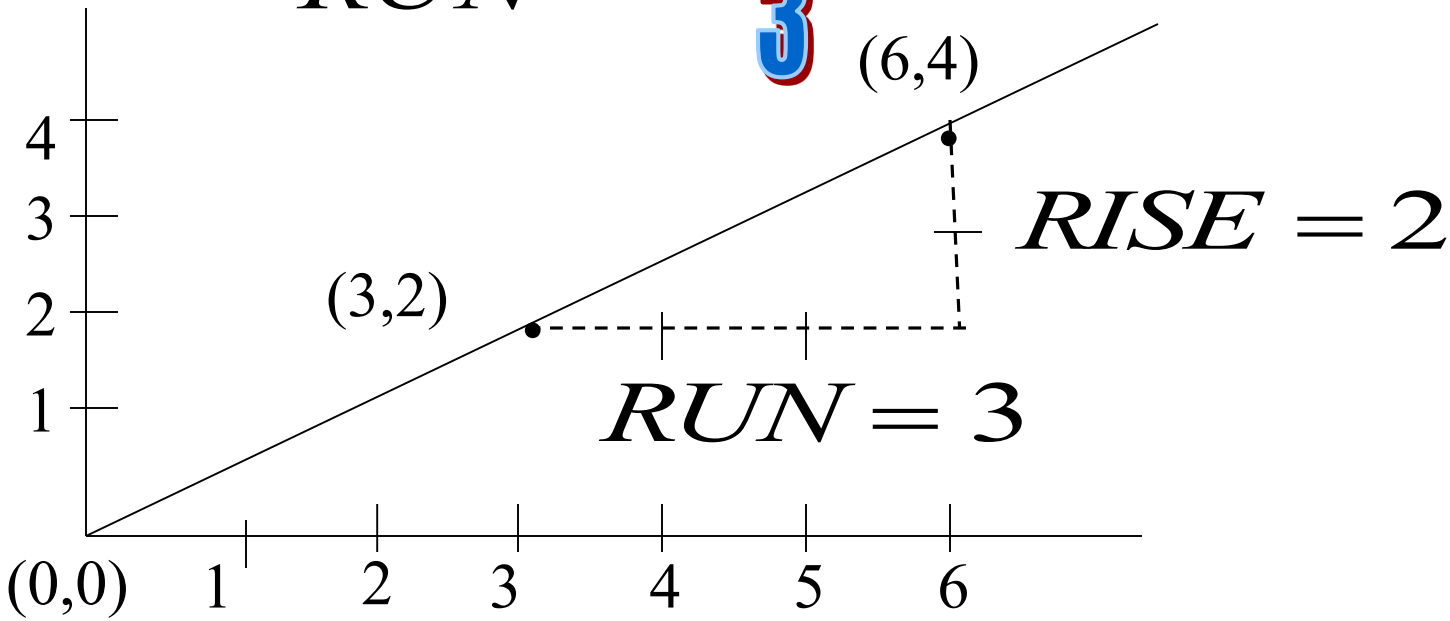
$$\text{SLOPE} = \frac{RISE}{RUN}$$



How much does this line rise?

How much does it run?

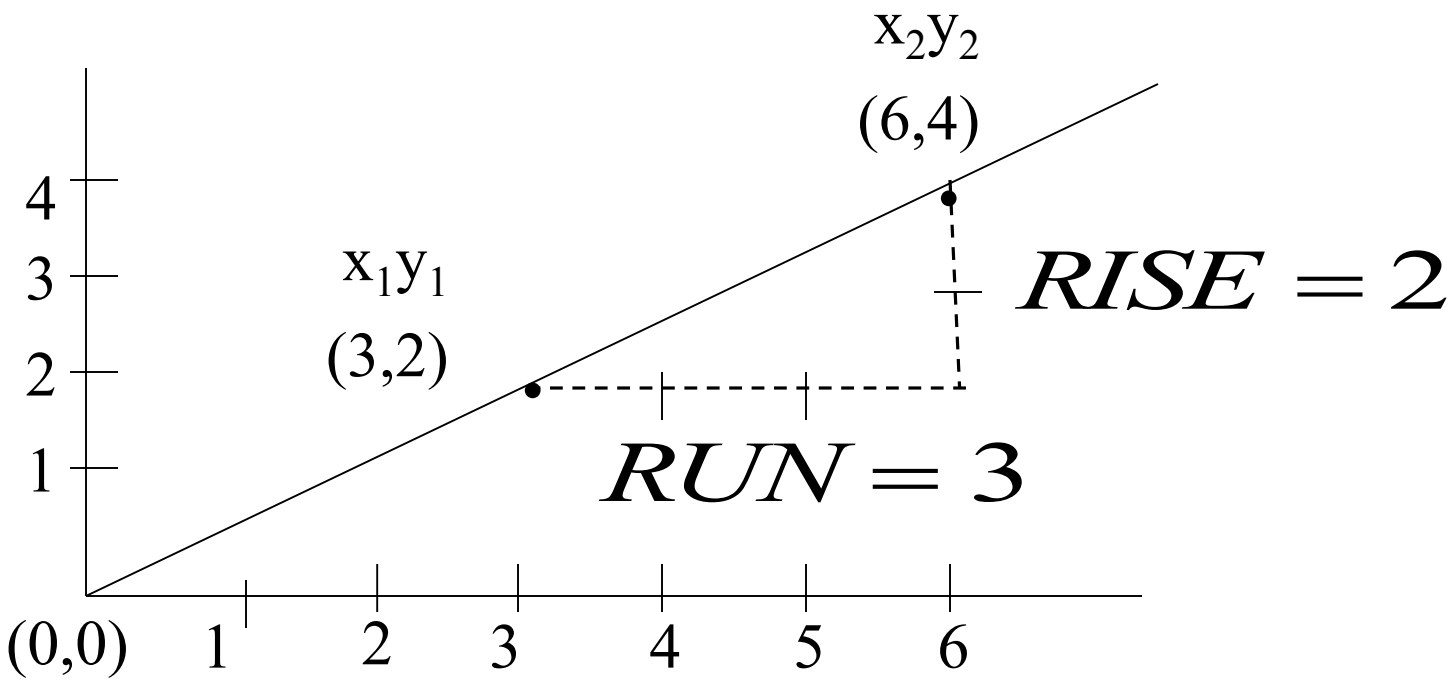
$$\text{SLOPE} = \frac{RISE}{RUN} = \frac{2}{3}$$



How much does this line rise? 2

How much does it run? 3

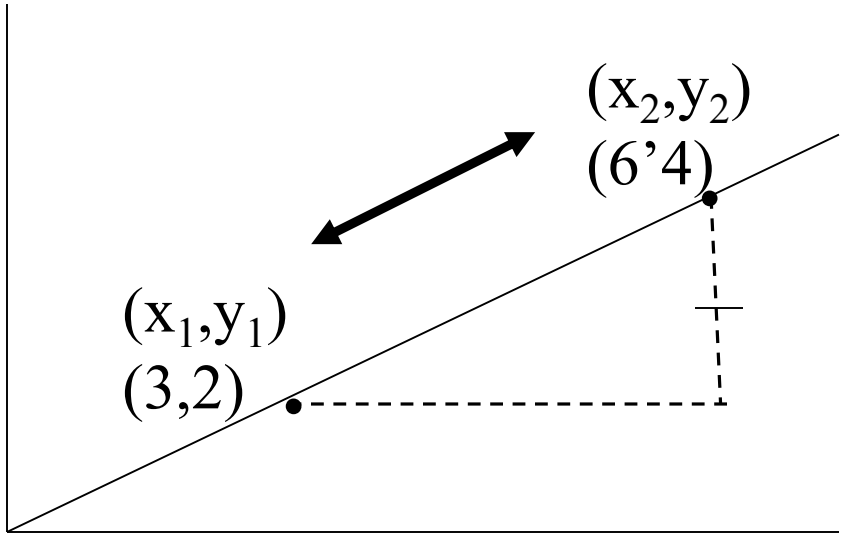
$$\text{m=SLOPE} = \frac{RISE}{RUN} = \frac{y_2 - y_1}{x_2 - x_1}$$

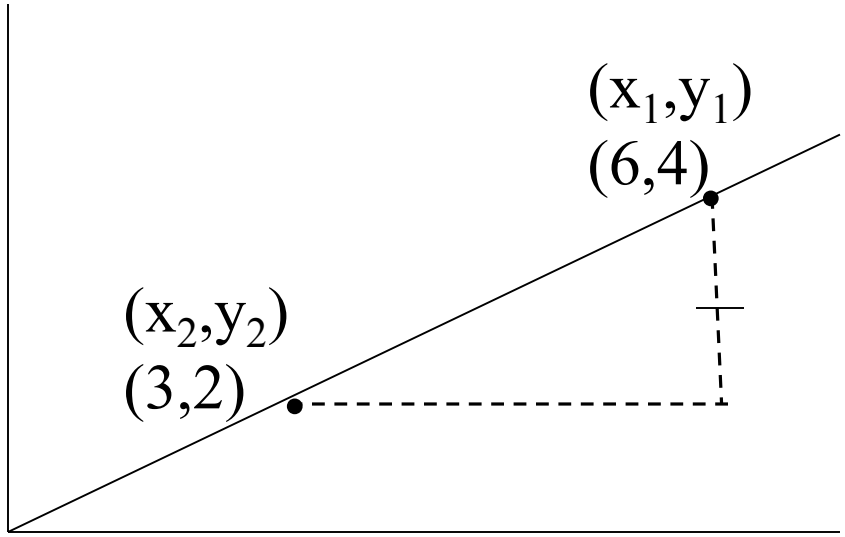


$$Slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{6-3} = \frac{2}{3}$$

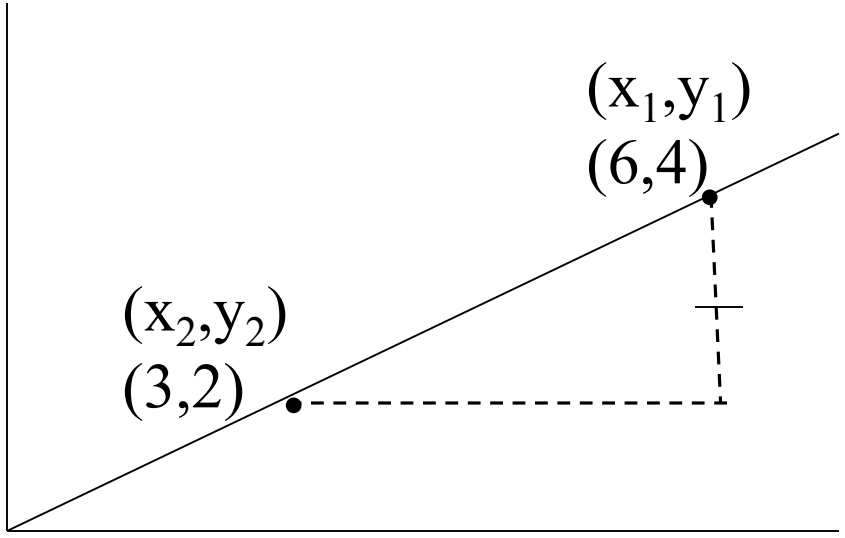
Switch points and calculate slope

Make (3,2) (x_2,y_2) & (6,4) (x_1,y_1)





Recalculation with points switched



$$Slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-6}{2-5} = \frac{-2}{-3} = \frac{2}{3}$$

Same slope as before

It doesn't matter what 2 points you choose on a line the slope must come out the same

Keeping Track of Signs When Finding The Slope Between 2 Points

- Be Neat & Careful
- Use (PARENTHASES)
- Double Check Your Work as you Go
- Follow 3 Steps

3 Steps for finding the Slope of a line between 2 Points (3,4)&(-2,6)

1st Step: Write x_1,y_1,x_2,y_2 over numbers

$$\begin{matrix} x_1 & y_1 & & x_2 & y_2 \\ (3,4) & \& & (-2,6) & & \end{matrix}$$

2nd Step: Write Formula and Substitute x_1,x_2,y_1,y_2 values.

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{-2-3}$$

3rd Step: Calculate & Simplify

$$\frac{6-4}{-2-3} = \frac{+2}{-5} = -\frac{2}{5}$$

Find the Slopes of Lines containing these 2 Points

1. (1,7) & (5,2)

2. (3,5) & (-2,-8)

3. (-3,-1) & (-5,-9)

4. (4,-2) & (-5,4)

5. (3,6) & (5,-5)

6. (1,-4) & (5,9)

ANSWERS

1. (1,7) & (5,2)

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-7}{5-1} = \frac{-5}{4}$$

2. (3,5) & (-2,-8)

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-5}{-2-3} = \frac{-13}{-5} = \frac{13}{5}$$

3. (-3,-1) & (-5,-9)

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9-(-1)}{-5-(-3)} = \frac{-8}{-2} = \frac{4}{1}$$

4. (4,-2) & (-5,4)

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{-5-4} = \frac{6}{-9} = -\frac{2}{3}$$

5. (3,6) & (5,-5)

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-6}{5-3} = \frac{-11}{2}$$

6. (1,-4) & (5,9)

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-(-4)}{5-1} = \frac{13}{4}$$

Solve for y if (9,y) & (-6,3) & m=2/3

$$Slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{2}{3} = \frac{3 - y_1}{-6 - 9} = \frac{3 - y}{-15}$$

$$(-15)\frac{2}{3} = \frac{3 - y_1}{-6 - 9} = \frac{3 - y}{-15}(-15)$$

$$(-5)2 = 3 - y$$

$$-10 = 3 - y$$

$$-13 = -y$$

$$13 = y$$

Review Finding the Slopes of Lines Given 2 Points

1st Step: Write x_1, x_2, y_1, y_2 over numbers

2nd Step: Write Formula and Substitute x_1, x_2, y_1, y_2 values.

3rd Step: Calculate & Simplify $m = Slope = \frac{y_2 - y_1}{x_2 - x_1}$

NOTE:

Be Neat, Careful, and Precise and Check your work as you go..

$$SLOPE = m = \frac{RISE}{RUN}$$

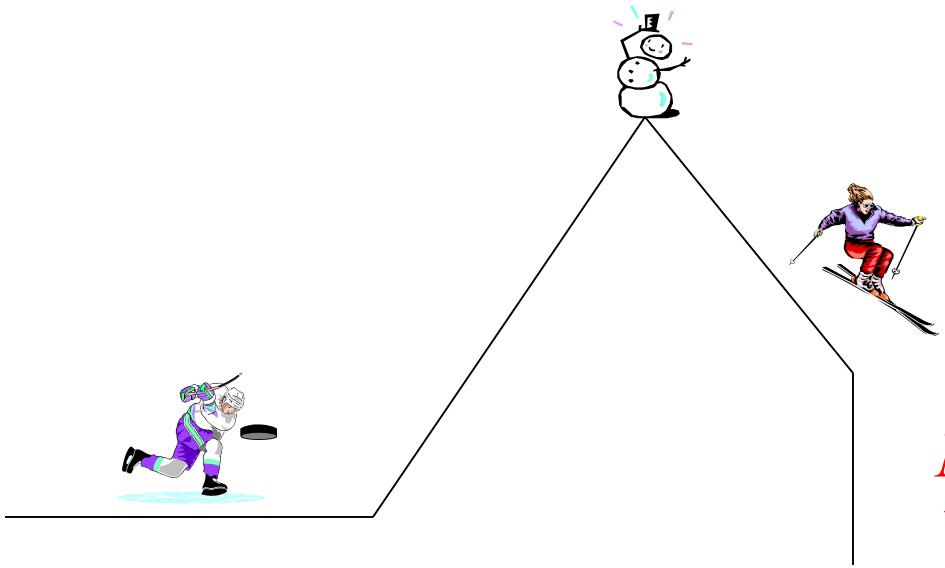
Positive Slope
Is Up the Hill

Negative Slope
Is Down the Hill

NO Slope
Vertical Drop

ZERO Slope Horizontal

$$SLOPE = m = \frac{RISE}{RUN}$$



NO Slope
Vertical Drop

ZERO Slope Horizontal

$$\frac{RISE}{RUN} = \frac{0}{any\_number} = 0$$

$$\frac{RISE}{RUN} = \frac{any\_number}{0} = Undefined(NO\_Slope)$$

Equations of a Line

There are ***3 Forms*** of Line Equations

- Standard Form: $ax+by=c$
- Slope Intercept Form: $y=mx+b$
- Point-Slope Form $y-y_1=m(x-x_1)$

All 3 describe the line completely but are used for different purposes. You can convert from one form to another.

Converting from Standard Form: ax+by=c to Slope Intercept Form

$$3x + 6y = 12$$

$$6y = -3x + 12$$

$$\frac{6}{6}y = \frac{-3}{6}x + \frac{12}{6}$$

JUST SOLVE FOR Y

$$y = -\frac{1}{2}x + 2$$

Slope Intercept Form: y=mx+b

Slope Intercept Form: y=mx+b

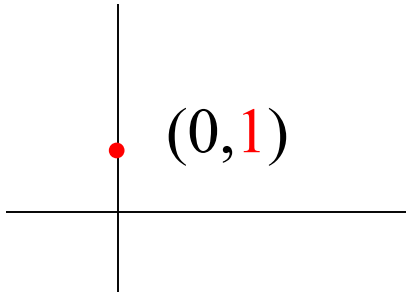
The great thing about this form is b is the y-intercept.

This makes graphing a line incredibly easy. Check it out. If

$$y = \frac{2}{3}x + 1$$

The y intercept is +1

Almost a free point on graph



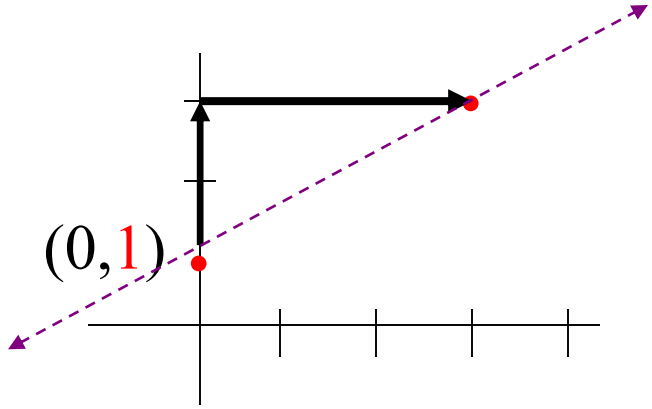
Slope Intercept Form: y=mx+b

All you have to do now is use the slope to rise and run

from the intercept & connect the points.

$$y = \frac{2}{3}x + 1$$

$$m = \frac{rise}{run} = \frac{2}{3}$$



Rise 2 and Run 3 from the y-intercept & connect points.

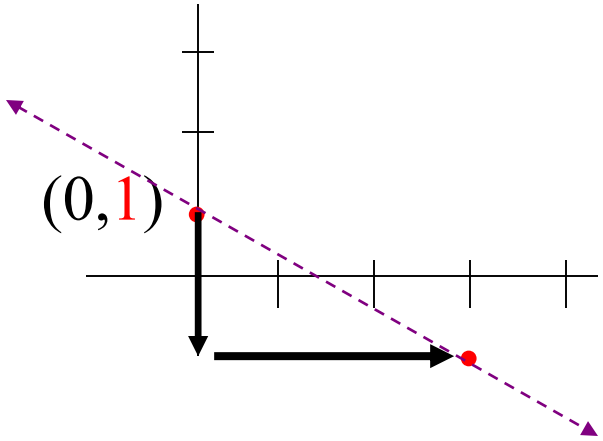
y=mx+b when m is negative

All you have to do now is use the slope to rise and run

from the intercept & connect the points.

$$y = -\frac{2}{3}x + 1$$

$$m = \frac{rise}{run} = -\frac{2}{3}$$



Rise -2 and Run 3 from the y-intercept & connect points.

Slope Intercept Form: y=mx+b
GRAPH THESE LINEAR EQUATIONS

Label y-intercept & Use one big graph

$$y = \frac{1}{2}x + 1 \qquad y = \frac{2}{5}x + 3$$

$$y = \frac{3}{2}x - 1 \qquad y = \frac{-1}{2}x + 1$$

If linear equation is not in y=mx+b form solve for y

$2y = 5x - 4$

Solution Steps to Solve for y:

$\frac{2}{2}y = \frac{5}{2}x - \frac{4}{2}$

Divide by 2

$y = \frac{5}{2}x - 2$

Now it is

This line has an y intercept of -2 and rises 5 and runs 2.

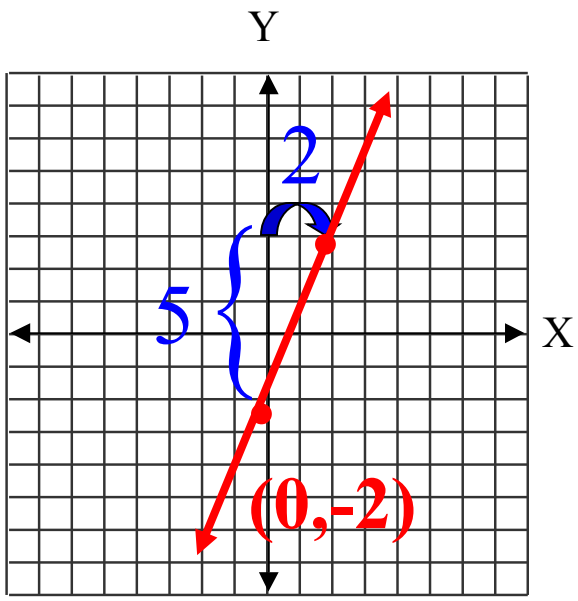
Graph $2y = 5x - 4$

Graphing a line with slope intercept equation

$$2y = 5x - 4$$

$$\frac{2}{2}y = \frac{5}{2}x - \frac{4}{2}$$

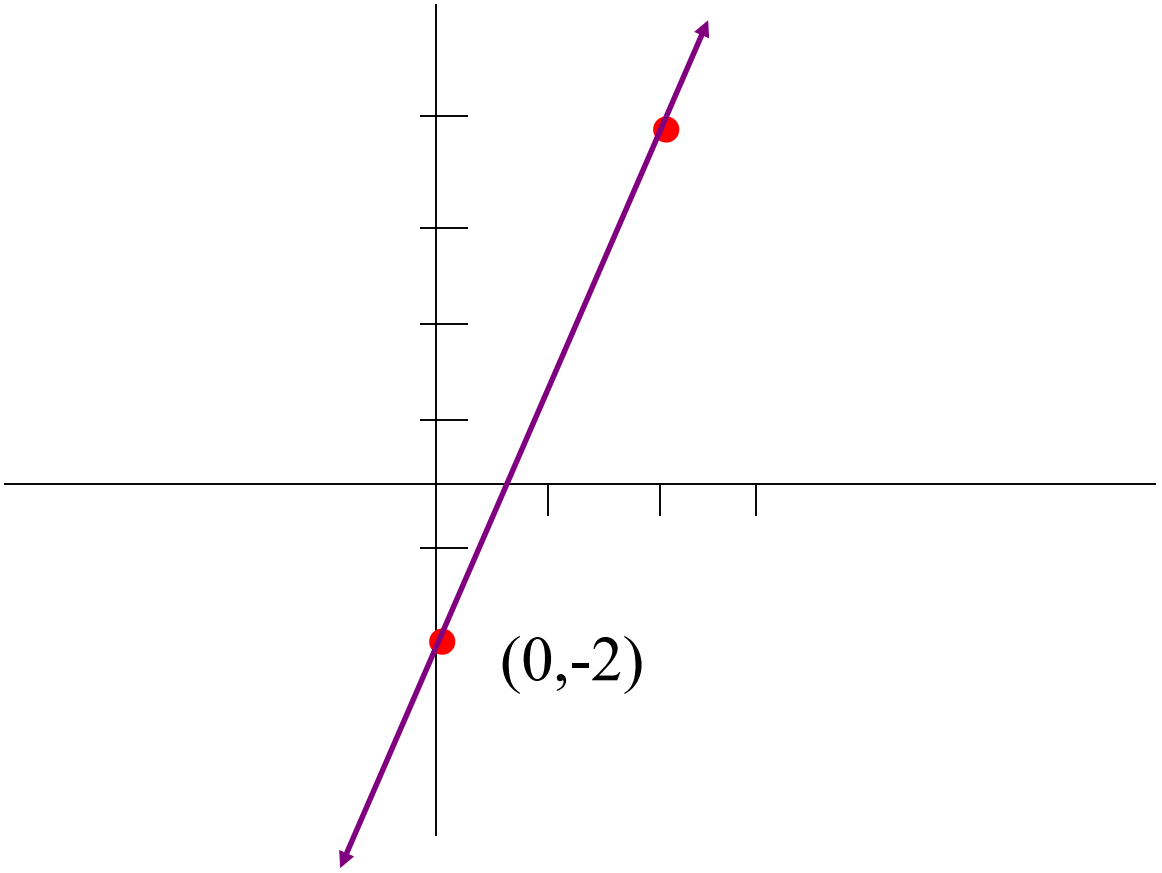
$$y = \frac{5}{2}x - 2$$



1. Solve for y:
2. Y-Intercept is 1st Point.
3. From the y-intercept
   Rise 5 and run 2 for
   Second Point.
4. Connect Points with line.

$y = \frac{5}{2}x - 2$

Now it is easy to graph



Put into slope-intercept form and graph

$$3y = 9x + 3$$

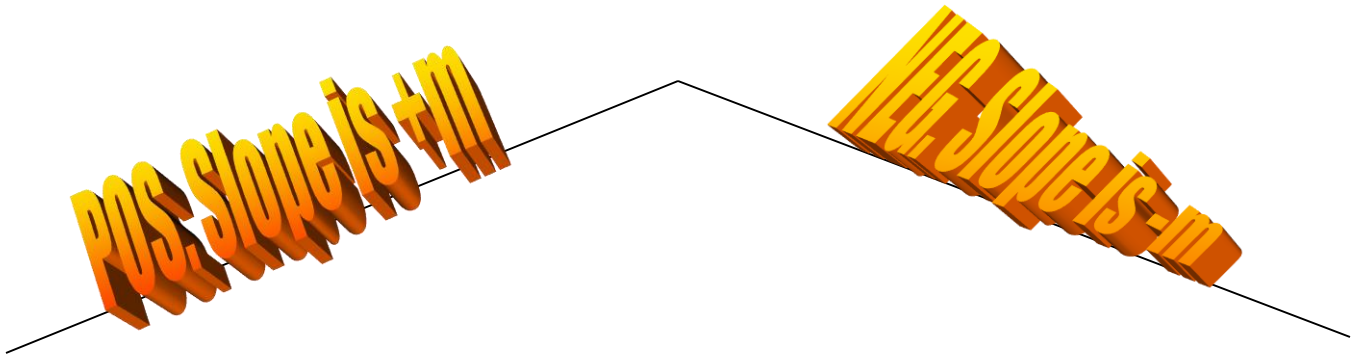
$$4y = 8x - 4$$

$$y - 5 = 6x$$

$$2y - 4 = 6x - 2$$

Review Steps of Graphing from the Slope Intercept Equation

1. Make sure equation is in y=mx+b form
2. Plot b(y-intercept) on graph (0,b)
3. From b, Rise and Run according to the slope to plot 2nd point.
4. Check sign of slope visually



Find the Equation of a Line (Given Pt. & Slope)

Given a point (2,5) & m=5 Write the Equation

$y = mx + b$

$5 = 5(2) + b$

$5 = 10 + b$

$-5 = b$

$y = 5x - 5$

1. Write Slope-Intercept Equation
2. 2. Plug-in (x,y) & m values
3. Solve for b
4. Plug m & b into Slope-Int. Eq.

Find the Equation of a Line (Given Pt. & Slope) Method 2 Using the Pt.-Slope Eq.

Given a point (2,5) & m=5 Write the Equation

$y - y_1 = m(x - x_1)$

$y - 5 = 5(x - 2)$

$y - 5 = 5x - 10$

$y = 5x - 5$

$y = 5x - 5$

1. Write Pt.-Slope Equation
2. 2. Plug-in (x,y) & m values
3. Solve for y

Find the Equation of a Line (Given 2 Points)

Given a point (x_1,y_1) & (x_2,y_2)

(2,5) & (3,10)

1. Find Slope using $m = Slope = \frac{y_2 - y_1}{x_2 - x_1}$
2. Write Slope-Intercept Equation
3. Plug-in (x,y) & m values
4. Solve for b

$y = mx + b$

$5 = 5(2) + b$

$5 = 10 + b$

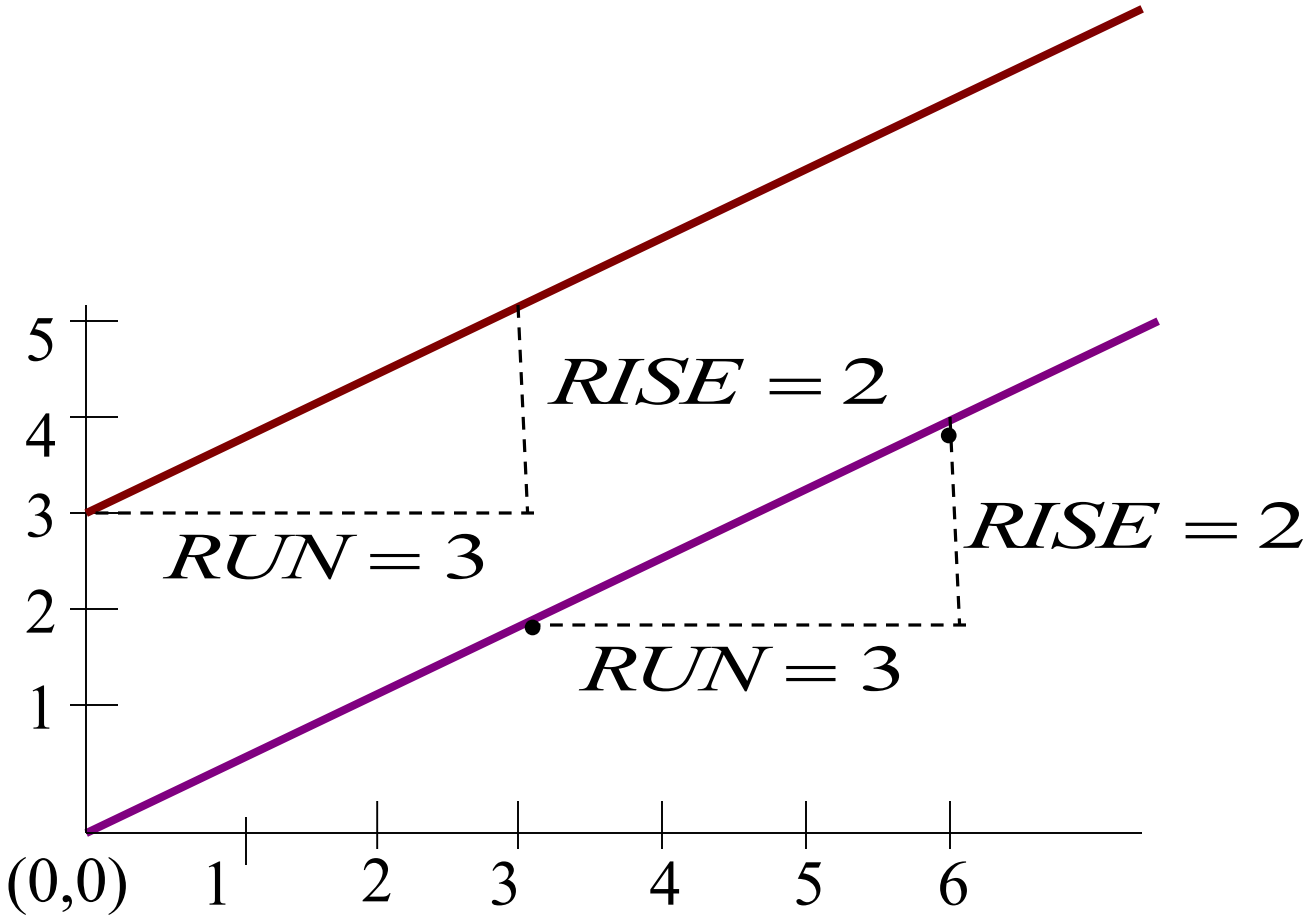
$-5 = b$

5. Plug m & b into Slope-Int. Eq.

$y = 5x - 5$

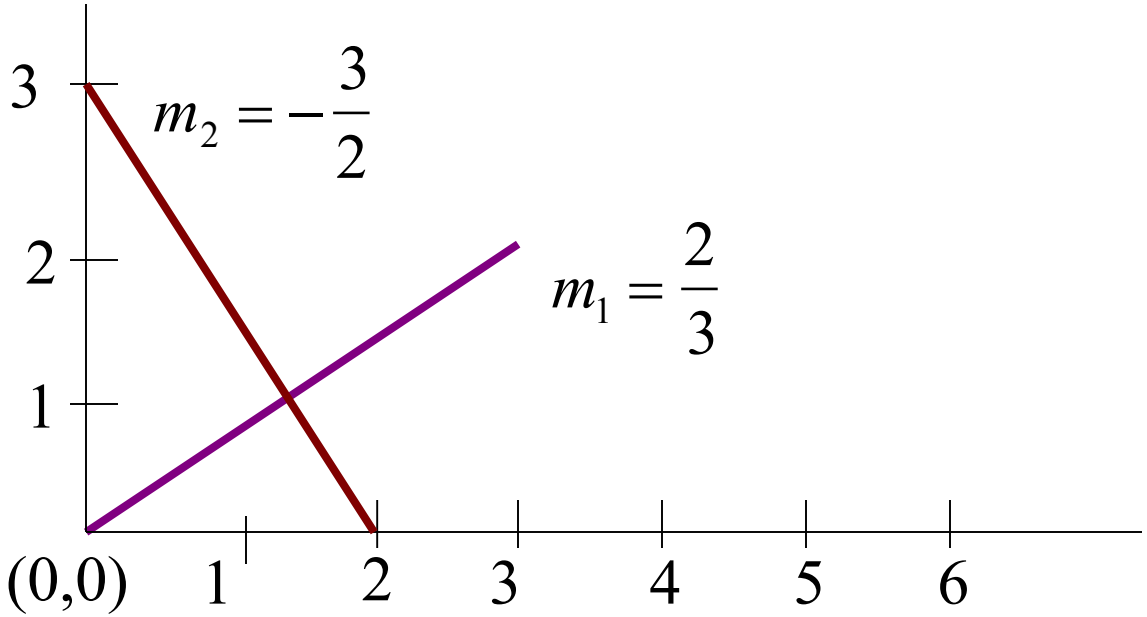
Parallel Lines

Have the Same Slope



Perpendicular Lines

Have Neg. Reciprocal Slopes



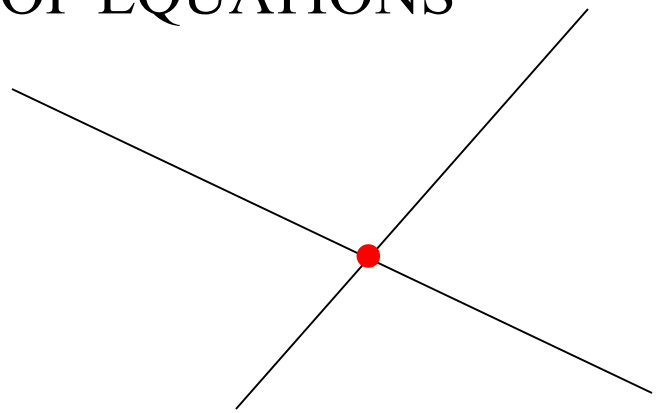
$$m_1 \bullet m_2 = \frac{2}{3} \bullet -\frac{3}{2} = -1$$

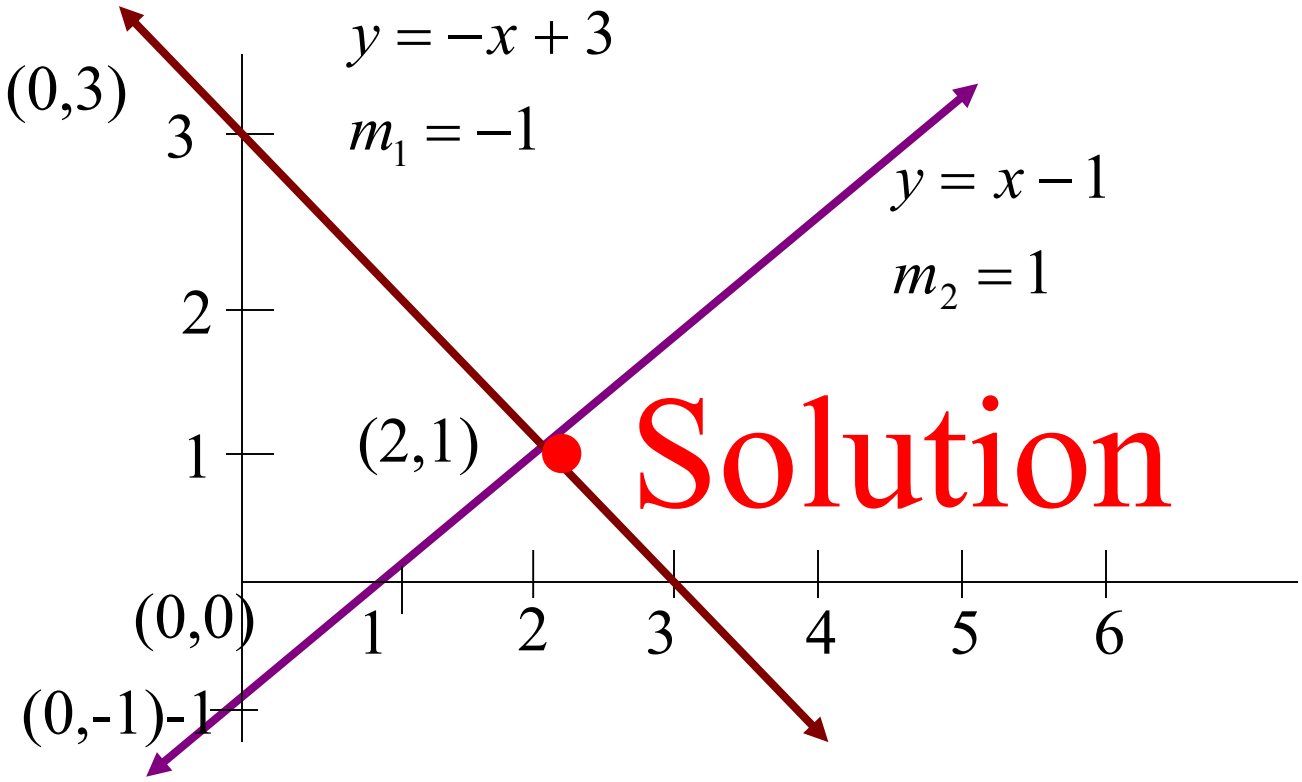
Systems of Equations

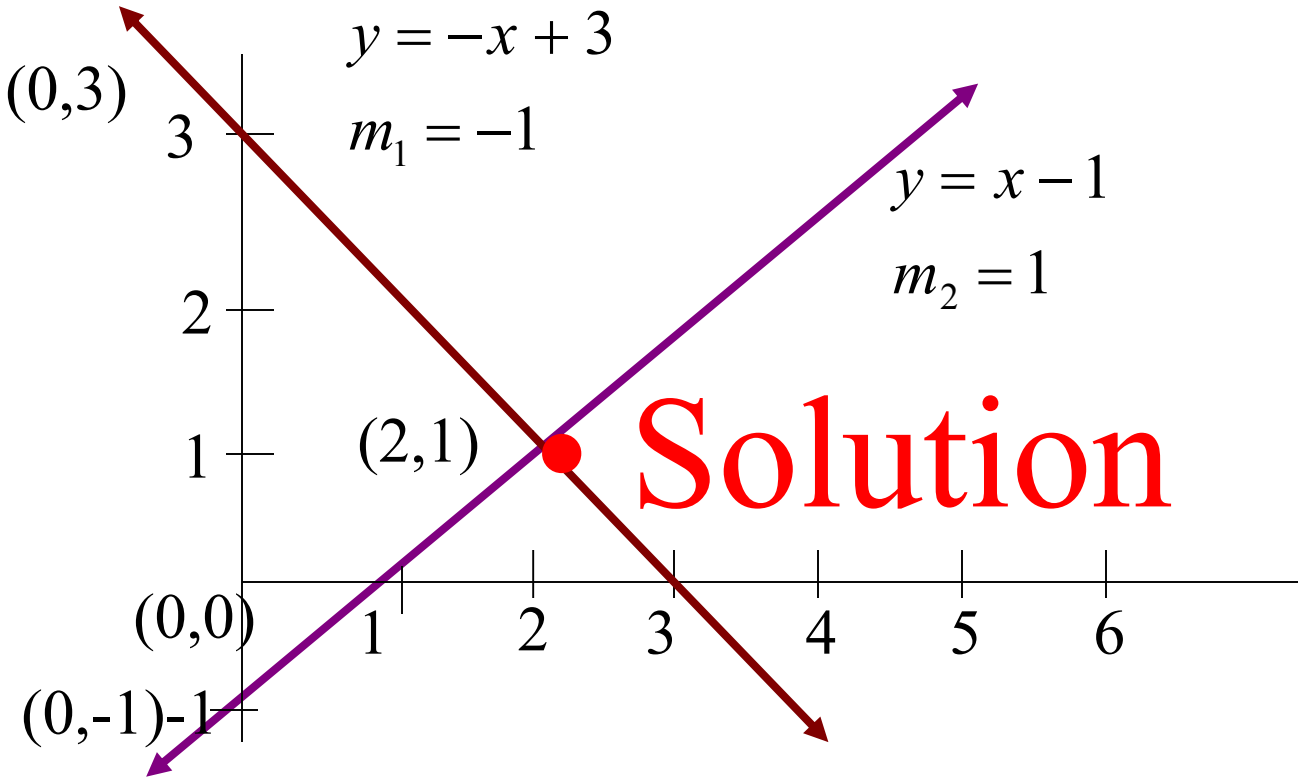
Given 2 linear equations

The single point where they intersect is a solution to either equation

It is also the solution to both equations or what we call the solution to the SYSTEM OF EQUATIONS




$y = -x + 3$
$m_1 = -1$
$y = x - 1$
$m_2 = 1$
(0,3)
3
2
1
(2,1)
Solution
(0,0)
1
2
3
4
5
6
(0,-1)
-1


$y = -x + 3$
$m_1 = -1$
$y = x - 1$
$m_2 = 1$
(0,3)
3
2
1
(2,1)
Solution
(0,0)
1
2
3
4
5
6
(0,-1)
-1

Systems of Equations

The Solution is where the two lines meet (or intersect)