3.4 Solving Equations with Variables on Both Sides

Goals • Collect variables on one side of an equation.
• Use equations to solve real-life problems.

VOCABULARY

Identity An identity is an equation that is true for all values of the variable.

Example 1   *Collect Variables on One Side*

Solve $4x - 10 = 32 - 3x$.

Solution

Look at the coefficients of the *x*-terms. Because 4 is greater than -3, collect the *x*-terms on the left side.

$4x - 10 = 32 - 3x$	**Write original equation.**
$4x - 10 + \underline{3x} = 32 - 3x + \underline{3x}$	**Add** $\underline{3x}$ **to each side.**
$\underline{7x} - 10 = 32$	**Simplify.**
$\underline{7x} - 10 + \underline{10} = 32 + \underline{10}$	**Add** $\underline{10}$ **to each side.**
$\underline{7x} = \underline{42}$	**Simplify.**
$\dfrac{7x}{7} = \dfrac{42}{7}$	**Divide each side by** $\underline{7}$.
$x = \underline{6}$	**Simplify.**

Check	$4x - 10 = 32 - 3x$	**Write original equation.**
	$4(\underline{6}) - 10 \stackrel{?}{=} 32 - 3(\underline{6})$	**Substitute** $\underline{6}$ **for *x*.**
	$\underline{14} = \underline{14}$	**Solution is** $\underline{correct}$.

Example 2 — *Many Solutions or No Solution*

Solve the equation.

a. $2(4x + 5) = 8x + 10$ **b.** $x - 1 = x + 7$

Solution

a. $2(4x + 5) = 8x + 10$ Write original equation.

  $\underline{8x + 10} = 8x + 10$ Use distributive property.

  $\underline{10} = \underline{10}$ Subtract $\underline{8x}$ from each side.

Answer All values of x $\underline{\text{are solutions}}$, because $\underline{10 = 10 \text{ is}}$ $\underline{\text{always true}}$. The original equation is $\underline{\text{an identity}}$.

b. $x - 1 = x + 7$ Write original equation.

  $\underline{-1 \neq 7}$ Subtract $\underline{x}$ from each side.

Answer The original equation $\underline{\text{has no solution}}$, because $\underline{-1 \neq 7 \text{ for any value of } x}$.

Example 3 — *Solving More Complicated Equations*

Solve $3(2 - x) + 2x = -5(x + 2)$.

Solution

  $3(2 - x) + 2x = -5(x + 2)$ Write original equation.

  $\underline{6 - 3x} + 2x = \underline{-5x - 10}$ Use distributive property.

  $\underline{6 - x} = \underline{-5x - 10}$ Combine like terms.

  $\underline{6 + 4x} = \underline{-10}$ Add $\underline{5x}$ to each side.

  $\underline{4x} = \underline{-16}$ Subtract $\underline{6}$ from each side.

  $x = \underline{-4}$ Divide each side by $\underline{4}$.

✔ *Checkpoint* **Solve the equation.**

1. $6x + 33 = 5x$

-33

2. $10y + 22 = 8y$

-11

3. $b = 9b - 24$

3

4. $-2n = 3n + 17$

$-3\frac{2}{5}$

5. $13m - 26 = 13m$

no solution

6. $-6(4 - 2x) = 12x - 24$

all real numbers

7. $15a - 2(4a + 5) = -6a$

$\frac{10}{13}$

8. $\frac{1}{4}(12 - 16q) = 5(q + 6)$

-3