Writing and Graphing Linear Equations

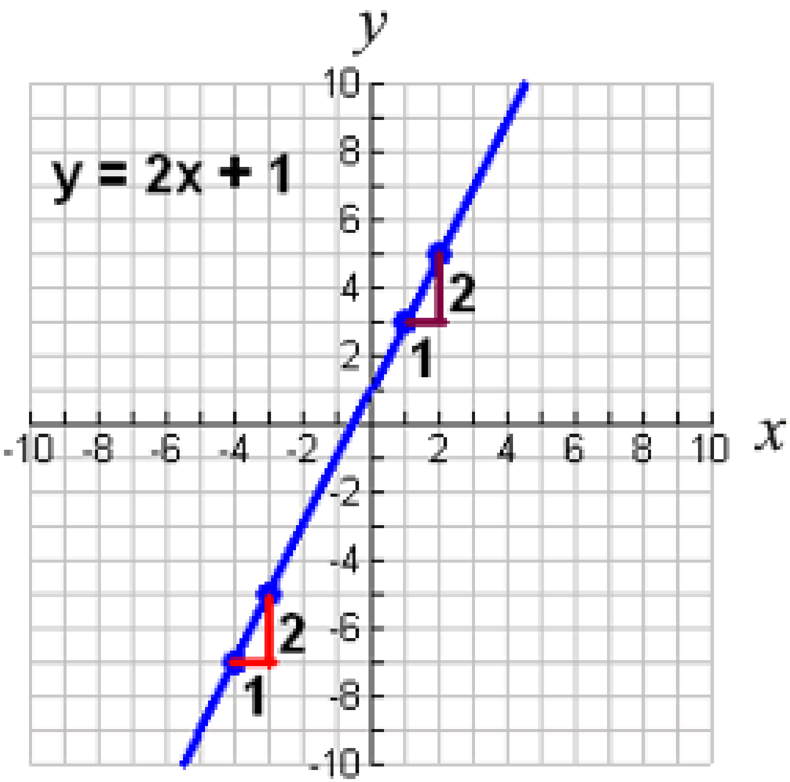
Linear equations can be used to represent relationships.

Writing Equations and Graphing

- These activities introduce rates of change and defines slope of a line as the ratio of the vertical change to the horizontal change.

- This leads to graphing a linear equation and writing the equation of a line in three different forms.

Linear equation – An equation whose solutions form a straight line on a coordinate plane.
Collinear – Points that lie on the same line.
Slope – A measure of the steepness of a line on a graph; rise divided by the run.

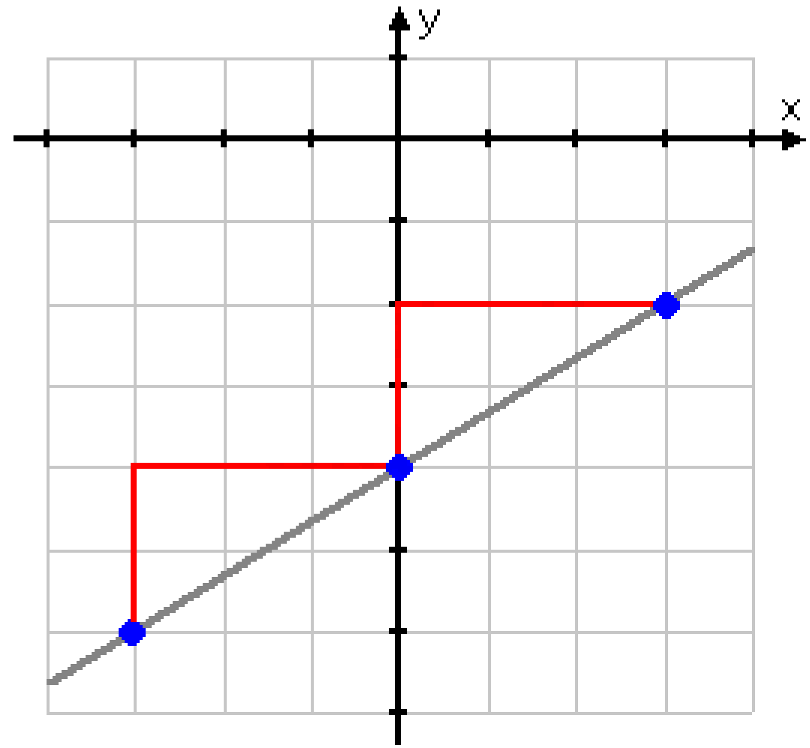


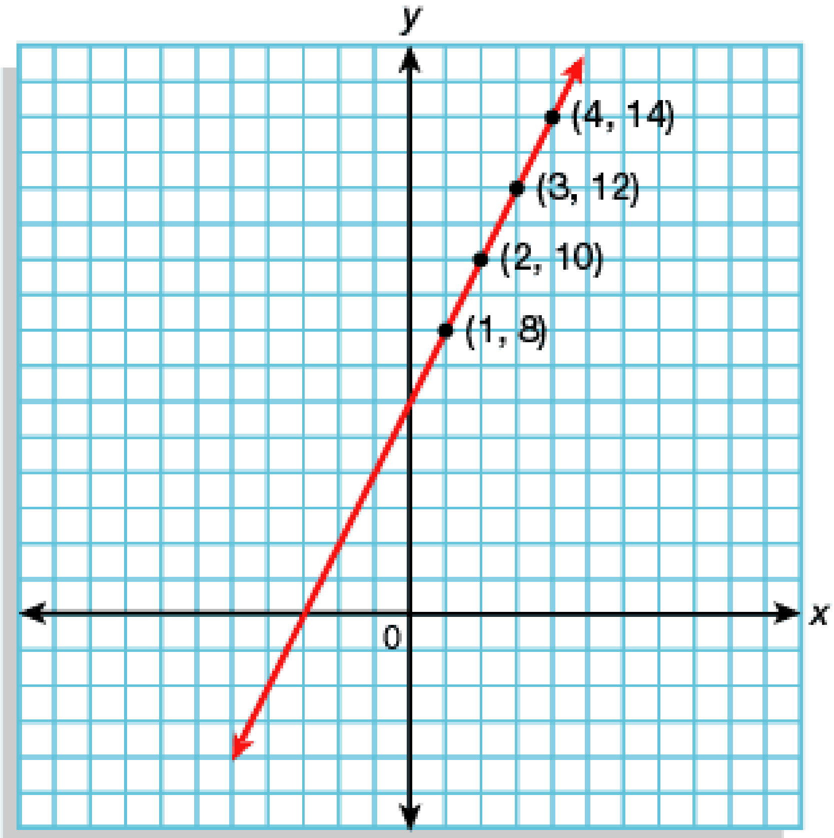
A ***linear equation*** is an equation whose solutions fall on a line on the coordinate plane. *All solutions* of a particular linear equation fall on the line, and all the points on the line are solutions of the equation.

Look at the graph to the left, points (1, 3) and (-3, -5) are found on the line and are solutions to the equation.

If an equation is *linear*, a constant change in the x-value produces a constant change in the y-value.

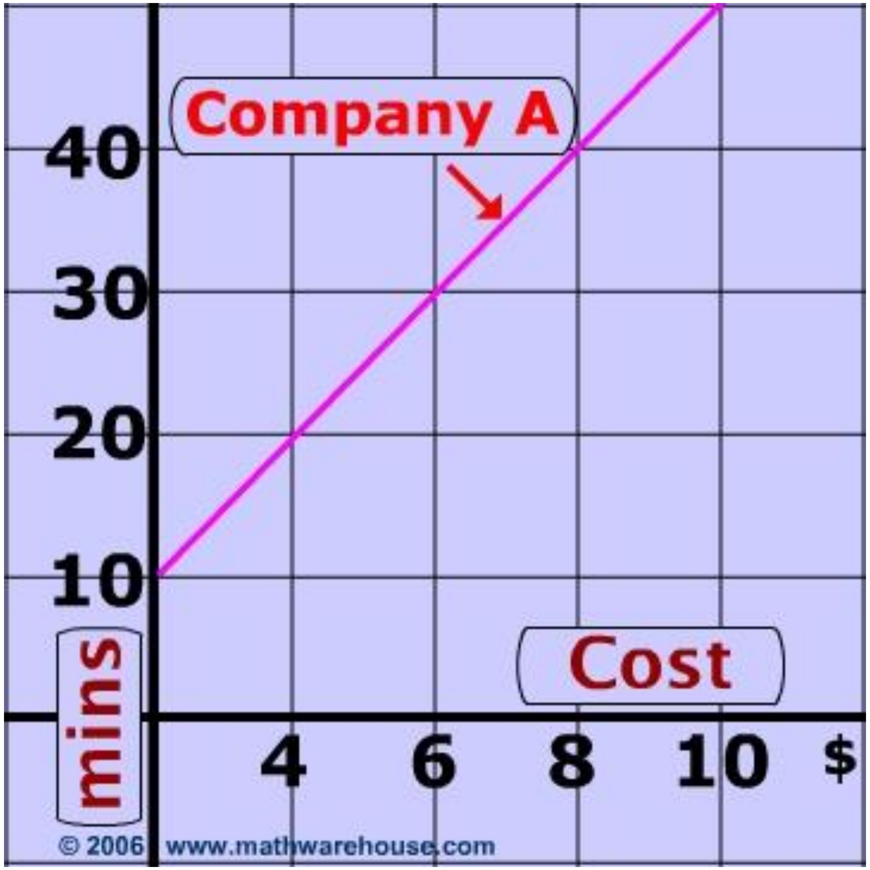
The graph to the right shows an example where each time the x-value increases by 2, the y-value increases by 3.





The equation
$y = 2x + 6$
is a *linear equation*
because it is the graph of a straight line and each time x increases by 1 unit, y increases by 2

X	Y = 2x + 6	Y	(x, y)
1	2(1) + 6	8	(1, 8)
2	2(2) + 6	10	(2, 10)
3	2(3) + 6	12	(3, 12)
4	2(4) + 6	14	(4, 14)
5	2(5) + 6	16	(5, 16)



Real world example

The graph
(c = 5x + 10)
at the left shows the cost for Company A cell phone charges.

What does Company A charge for 20 minutes of service?

Graphing equations can be down several different ways. Tables can be used to graph linear equations by simply graphing the points from the table.

Missing Values

Name ______________________________

A piece of paper with an incomplete table showing number pairs was left on a student's desk after a mathematics class. Using what you know about patterns, answer these questions:

x	*y*
2	_____
3	_____
4	16
5	20
6	24
7	_____
8	32

1. Predict the missing values for *y*, and record them in the table. Describe how you made your predictions.

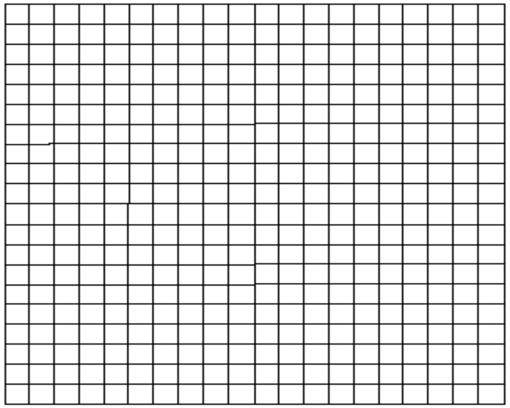
   ____

   ____

2. Graph the data in the table on the coordinate grid below.
3. Describe a general rule to help you determine the value for *y* if you are given the value for *x*.

   ____

4. Use your rule to find the value for *y* when $x = -1$. Show this point on the line you graphed.



Complete the table below, then graph and tell whether it is linear.

x	y = 2x + 3	y	(x, y)
-2			
-1			
0			
1			
2			

Can you determine if the equation is linear? The equation y = 2x + 3 is a linear equation because it is the graph of a straight line. Each time *x* increases by 1 unit, *y* increases by 2.

X	y = 2x + 3	y	(x,y)
-2	2 (-2) + 3	-1	(-2, 1)
-1	2(-1) + 3	1	(-1, 1)
0	2(0) + 3	3	(0, 3)
1	2(1) + 3	5	(1, 5)
2	2(2) + 3	7	(2, 7)

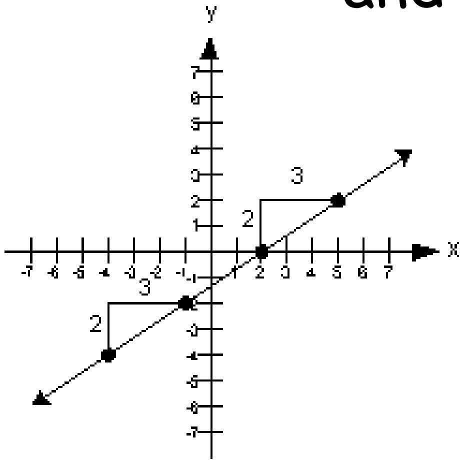
Slope

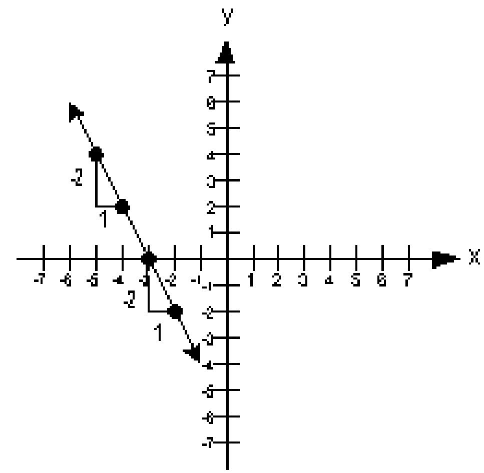
Rate of change

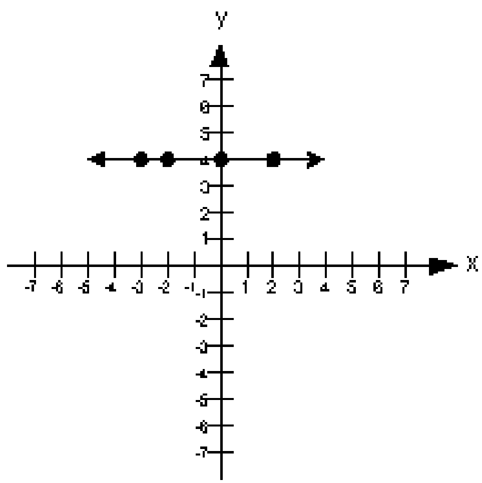
Slope of a line is its rate of change. The following example describes how slope (rate of change) is applied.
Rate of change is also know as ***grade*** or ***pitch***, or ***rise over run***. Change is often symbolized in mathematics by a delta for which the symbol is the Greek letter: Δ

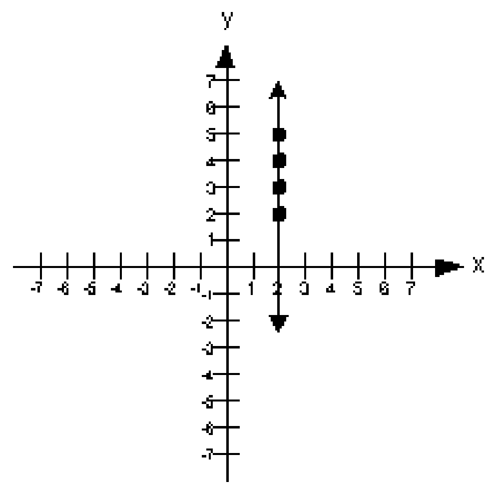
$$\textbf{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y-coordinates}}{\text{change in x-coordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Finding slope (rate of change) using a graph and two points.

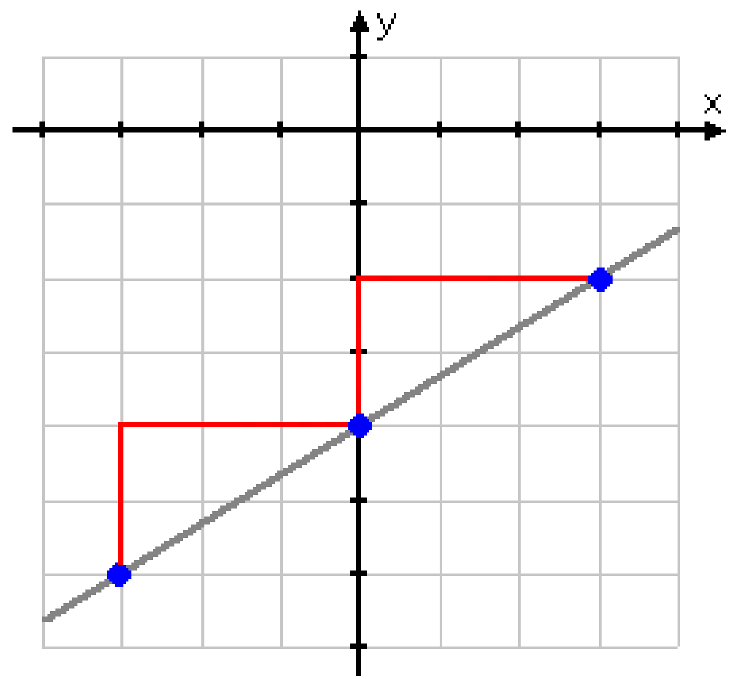




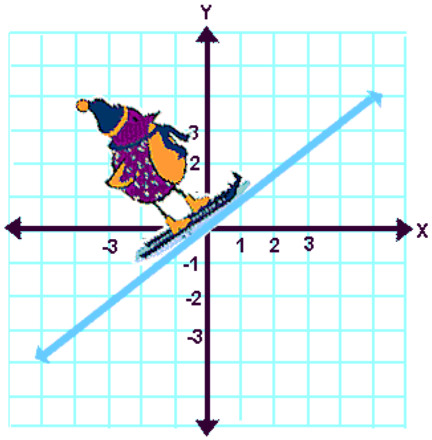


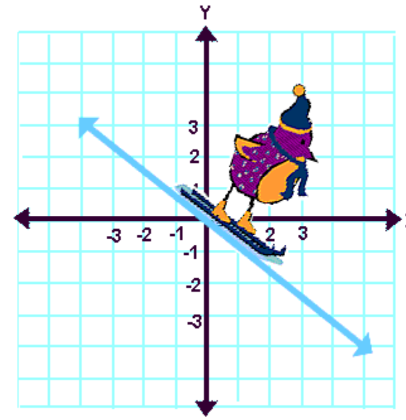


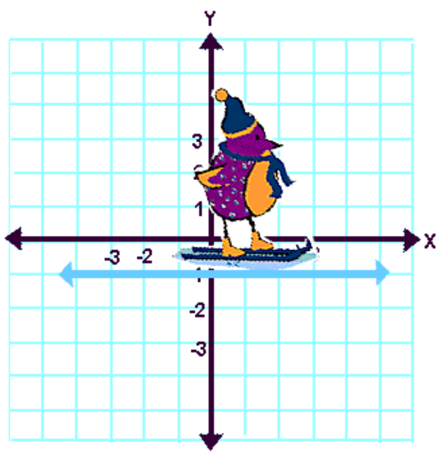
If an equation is linear, a constant change in the x-value corresponds to a constant change in the y-value. The graph shows an example where each time the x-value increases by 3, the y-value increases by 2.

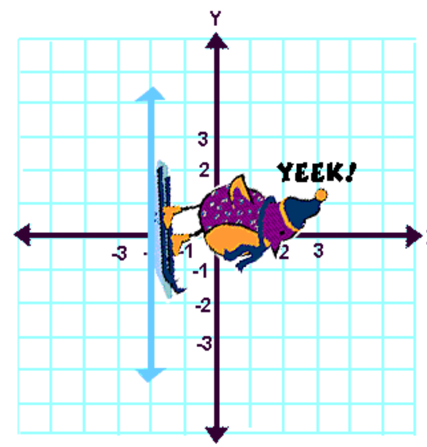


Slopes: positive, negative, no slope (zero), undefined.









Remember, linear equations have constant slope. For a line on the coordinate plane, slope is the following ratio. This ratio is often referred to as "rise over run".

Slope of a line

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Find the *slope* of the line that passes through each pair of points.

1) (1, 3) and (2, 4)

2) (0, 0) and (6, -3)

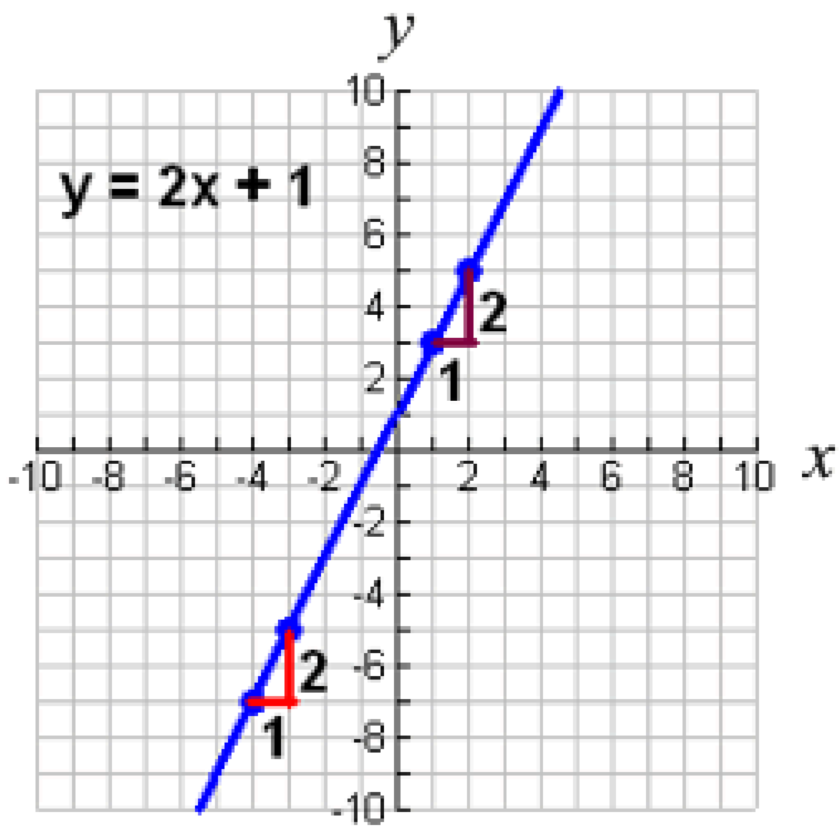
3) (2, -5) and (1, -2)

4) (3, 1) and (0, 3)

5) (-2, -8) and (1, 4)

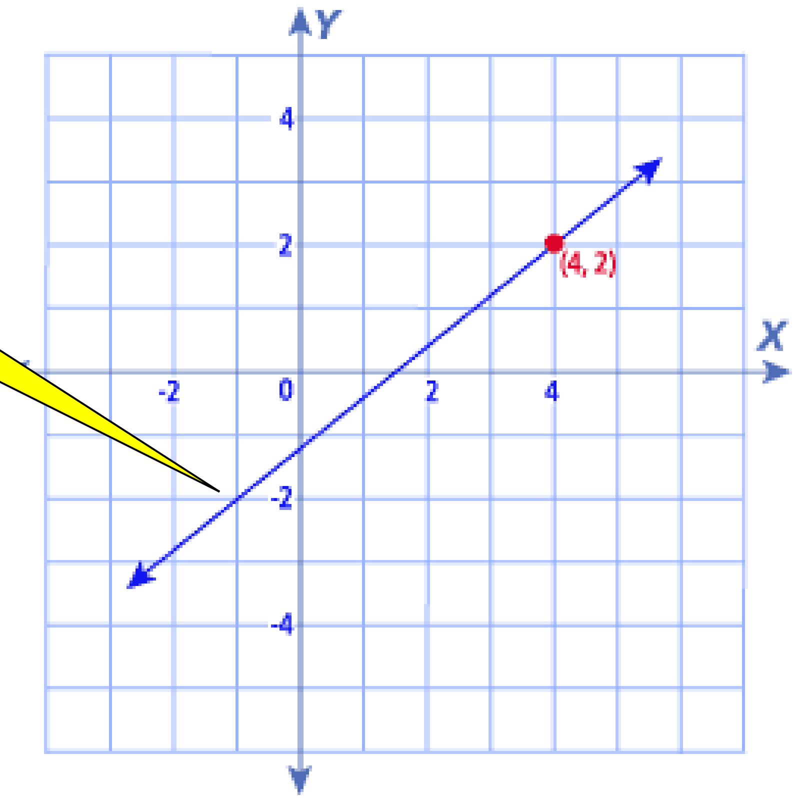
Graphing a Line Using a Point and the Slope

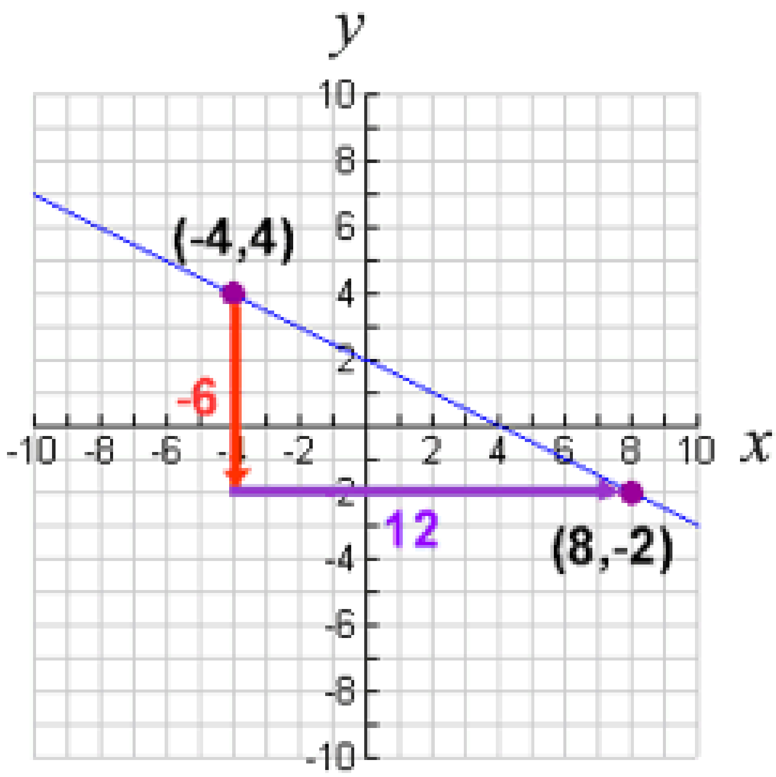
Graph the line passing through (1, 3) with slope 2.



Given the point (4, 2), find the slope of this line?

To make finding slope easier, find where the line crosses at an ***x*** and ***y*** junction.

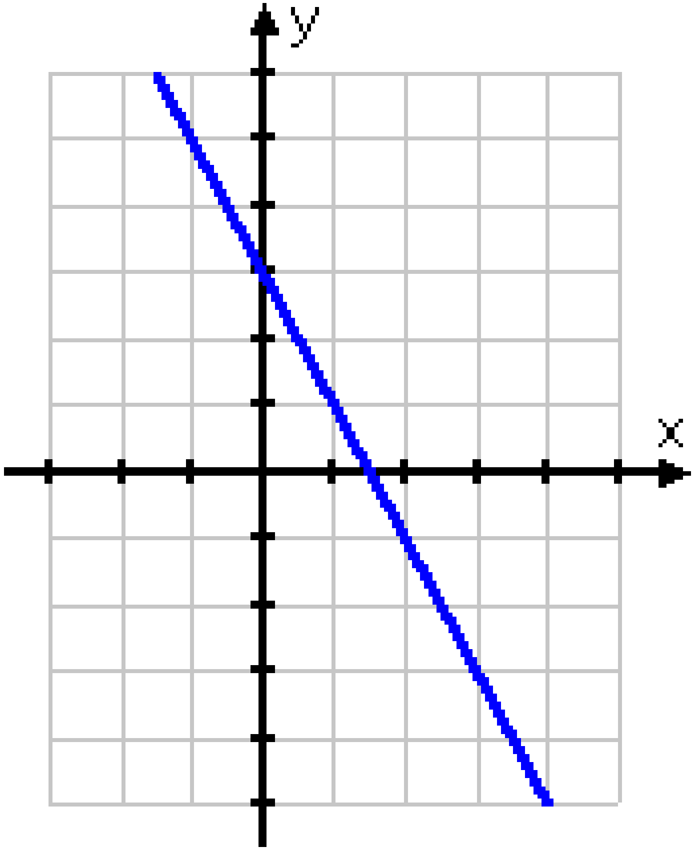




Finding Slope from a Graph

Use the graph of the line to determine its slope. Choose two points on the line (-4, 4) and (8, -2). Count the rise over run or you can use the slope formula. Notice if you switch (x_1, y_1) and (x_2, y_2), you get the same slope:

Use the graph to find the *slope* of the line.



Using Slopes and Intercepts

x-intercepts and y-intercepts

x-intercept – the x-coordinate of the point where the graph of a line crosses the x-axis (where $y = 0$).

y-intercept – the y-coordinate of the point where the graph of a line crosses the y-axis (where $x = 0$).

Slope-intercept form (of an equation) – a linear equation written in the form $y = mx + b$, where *m* represents slope and b represents the *y*-intercept.

Standard form (of an equation) – an equation written in the form of $Ax + By = C$, where A, B, and C are real numbers, and A and B are both $\neq 0$.

Standard Form of an Equation

- The standard form of a linear equation, you can use the x- and y-intercepts to make a graph.
- The x-intercept is the x-value of the point where the line crosses.
- The y-intercept is the y-value of the point where the line crosses.

$$Ax + By = C$$

To graph a linear equation in ***standard form***, you fine the x-intercept by substituting 0 for *y* and solving for *x*. Then substitute 0 for *x* and solve for *y*.

$$2x + 3y = 6$$
$$2x + 3(0) = 6$$
$$2x = 6$$
$$x = 3$$

The x-intercept is 3.
(y = 0)

$$2x + 3y = 6$$
$$2(0) + 3y = 6$$
$$3y = 6$$
$$y = 2$$

The y-intercept is 2.
(x = 0)

Let's take a look at that equation again!

$$2x + 3y = 6$$

Since 3(0) = 0, just cover up the 3y and solve what's left.

$$2x = 6$$

$$x = 3$$

$$2x + 3y = 6$$

Again, since 2(0) = 0, just cover up 2x and solve what's left.

$$3y = 6$$

$$y = 2$$

Since you are substituting (0) in for one variable and solving for the other, any number multiplied times (0) = 0.

So, in the first example 3(0) = 0, and in the second example 2(0) = 0.

Find the *x-intercept* and *y-intercept* of each line. Use the intercepts to graph the equation.

1) $x - y = 5$

2) $2x + 3y = 12$

3) $4x = 12 + 3y$

4) $2x + y = 7$

5) $2y = 20 - 4x$

Slope-intercept Form

$$y = mx + b$$

Slope-intercept Form

- An equation whose graph is a straight line is a ***linear equation***. Since a function rule is an equation, a function can also be linear.
- m = slope
- b = y-intercept

$$y = mx + b$$

(if you know the slope and where the line crosses the y-axis, use this form)

For example in the equation;

$y = 3x + 6$

$m = 3$, so the slope is 3

$b = +6$, so the y-intercept is +6

Let's look at another:

$y = 4/5x - 7$

$m = 4/5$, so the slope is 4/5

$b = -7$, so the y-intercept is -7

Please note that in the slope-intercept formula;

$y = mx + b$

the "y" term is all by itself on the left side of the equation.

That is very important!

WHY?

If the "y" is not all by itself, then we must first use the *rules of algebra* to isolate the "y" term. For example in the equation:

$$2y = 8x + 10$$

You will notice that in order to get "y" all by itself we have to divide both sides by 2.

After you have done that, the equation becomes:

$$y = 4x + 5$$

Only then can we determine the slope (4), and the y-intercept (+5)

OK…getting back to the lesson…
Your job is to *write the equation of a line* after you are given the *slope* and *y-intercept*…

Let's try one…

Given "m" (the slope remember!) = 2

And "b" (the y-intercept) = +9

All you have to do is plug those values into

$y = mx + b$

The equation becomes…

$y = 2x + 9$

Let's do a couple more to make sure you are expert at this.

Given m = 2/3, b = -12,

Write the equation of a line in slope-intercept form.

$y = mx + b$

$y = 2/3x - 12$

****

One last example…

Given m = -5, b = -1

Write the equation of a line in slope-intercept form.

$y = mx + b$

$y = -5x - 1$

Given the slope and y-intercept, write the equation of a line in *slope-intercept* form.

Slope-intercept form of an equation
$Y = mx + b$

1) $m = 3$, $b = -14$

2) $m = -\frac{1}{2}$, $b = 4$

3) $m = -3$, $b = -7$

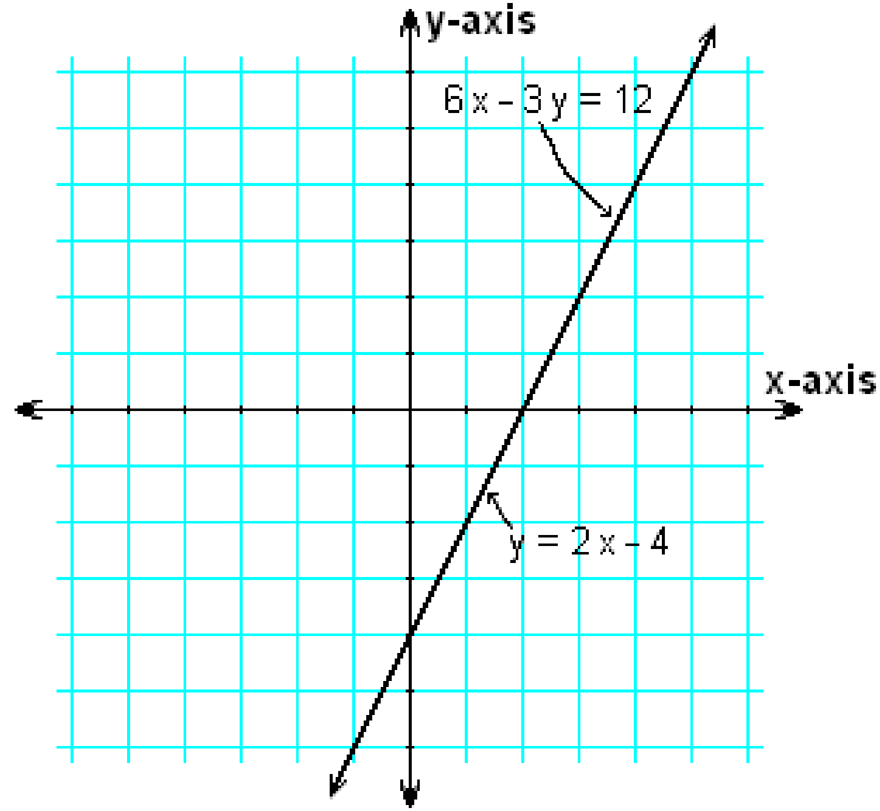
4) $m = 1/2$, $b = 0$

5) $m = 2$, $b = 4$

Using *slope-intercept* form to find slopes and y-intercepts

The graph at the right shows the equation of a line both in standard form and slope-intercept form.

You must rewrite the equation $6x - 3y = 12$ in slope-intercept to be able to identify the slope and y-intercept.



Using *slope-intercept* form to write equations, Rewrite the equation solving for $y =$ to determine the slope and y-intercept.

$3x - y = 14$

$\frac{-y}{-1} = \frac{-3x}{-1} + \frac{14}{-1}$

$y = 3x - 14$ or

$3x - y = 14$

$3x = y + 14$

$3x - 14 = y$

$x + 2y = 8$

$\frac{2y}{2} = \frac{-x}{2} + \frac{8}{2}$

$y = \frac{-1x}{2} + 4$

Write each equation in *slope-intercept* form.
Identify the slope and y-intercept.

$2x + y = 10$

$-4x + y = 6$

$4x + 3y = 9$

$2x + y = 3$

$5y = 3x$

Write the equation of a line in *slope-intercept* form that passes through points (3, -4) and (-1, 4).

Do you remember the slope formula?

1) Find the slope.

$$\frac{4 - (-4)}{-1 - 3} \quad \frac{8}{-4}$$

m = -2

2) Choose either point and substitute. Solve for b.

y = mx + b (3, -4)

-4 = (-2)(3) + b

-4 = -6 + b

2 = b

Substitute m and b in equation.

Y = mx + b

Y = -2x + 2

Write the equation of the line in *slope-intercept* form that passes through each pair of points.

1) (-1, -6) and (2, 6)

2) (0, 5) and (3, 1)

3) (3, 5) and (6, 6)

4) (0, -7) and (4, 25)

5) (-1, 1) and (3, -3)

Point-Slope Form

Writing an equation when you know a point and the slope

Point-Slope Form and Writing Equations

- Suppose you know that a line passes through the point (3, 4) with slope 2. You can quickly write an equation of the line using the x- and y-coordinates of the point and using the slope.
- The ***point-slope*** form of the equation of a nonvertical line that passes through the (x_1, y_1) with slope m.

$$y - y_1 = m(x - x_1)$$

(if you know a point and the slope, use this form)

Let's try a couple.

Using ***point-slope*** form, write the equation of a line that passes through (4, 1) with slope -2.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -2(x - 4)$$ Substitute 4 for x_1, 1 for y_1 and -2 for m.

Write in ***slope-intercept*** form.

$$y - 1 = -2x + 8$$ Add 1 to both sides

$$y = -2x + 9$$

One last example

Using ***point-slope*** form, write the equation of a line that passes through (-1, 3) with slope 7.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 7[x - (-1)]$$

$$y - 3 = 7(x + 1)$$

Write in ***slope-intercept*** form

$$y - 3 = 7x + 7$$

$$y = 7x + 10$$

If you know two points on a line, first use them to find the slope. Then you can write an equation using either point.

- Step one – Find the slope of a line with points (-4, 3), (-2, 1)

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

$$\frac{1-3}{-2-(-4)} = \frac{-2}{2} = -1$$

Step Two – Use either point to write the equation in ***point-slope*** form. Use (-4, 3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -1[x - (-4)]$$

$$y - 3 = -1(x + 4)$$

Write in ***slope-intercept*** form

$$y - 3 = -1(x + 4)$$

$$y - 3 = -x - 4$$

$$y = -x - 1$$

Equation Forms (review)

When working with straight lines, there are often many ways to arrive at an equation or a graph.

Slope Intercept Form

If you know the slope and where the line crosses the y-axis, use this form.

$$y = mx + b$$

m = slope

b = y-intercept

(where the line crosses the y-axis)

Point Slope Form

If you know a point and the slope, use this form.

$$y - y_1 = m(x - x_1)$$

m = slope

(x_1, y_1) = a point on the line

Horizontal Lines

$y = 3$ (or any number)

Lines that are horizontal have a slope of zero. They have "run" but no "rise". The rise/run formula for slope always equals zero since rise = 0.

$$y = mx + b$$

$$y = 0x + 3$$

$$y = 3$$

This equation also describes what is happening to the y-coordinates on the line. In this case, they are always 3.

Vertical Lines

$x = -2$

Lines that are vertical have no slope

(it does not exist).

They have "rise", but no "run". The rise/run formula for slope always has a zero denominator and is undefined.

These lines are described by what is happening to their x-coordinates. In this example, the x-coordinates are always equal to -2.

There are several ways to graph a straight line given its equation.

Let's quickly refresh our memories on equations of straight lines:

Slope-intercept	Point-slope	Horizontal line	Vertical line
$y = mx + b$ When stated in "y=" form, it quickly gives the slope, m, and where the line crosses the y-axis, b, called the y-intercept.	$y - y_1 = m(x - x_1)$ when graphing, put this equation into "y=" form to easily read graphing information.	Y = 3 (or any #) Horizontal lines have a slope of zero – they have "run", but no "rise" – all of the y values are 3.	X = -2 (or any #) Vertical line have no slope (it does not exist) – they have "rise", but no "run" – all of the x values are -2.

Remember

If a point lies on a line, its coordinates make the equation true.

(2, 1) on the line

y = 2x -3 because

1 = 2(2) - 3

Before graphing a line, be sure that your equation starts with

" y ="

To graph 6x + 2y = 8 rewrite the equation:

2y = -6x + 8

Y = -3x + 4

Now graph the line using either slope intercept method or table method.

Practice with Equations of Lines

Answer the following questions dealing with equations and graphs of straight lines.

1) Which of the following equations passes through the points (2, 1) and (5, -2)?

a. $y = 3/7x + 5$

b. $y = -x + 3$

c. $y = -x + 2$

d. $y = -1/3x + 3$

2) Does the graph of the straight line with slope of 2 and y-intercept of 3 pass through the point (5, 13)?

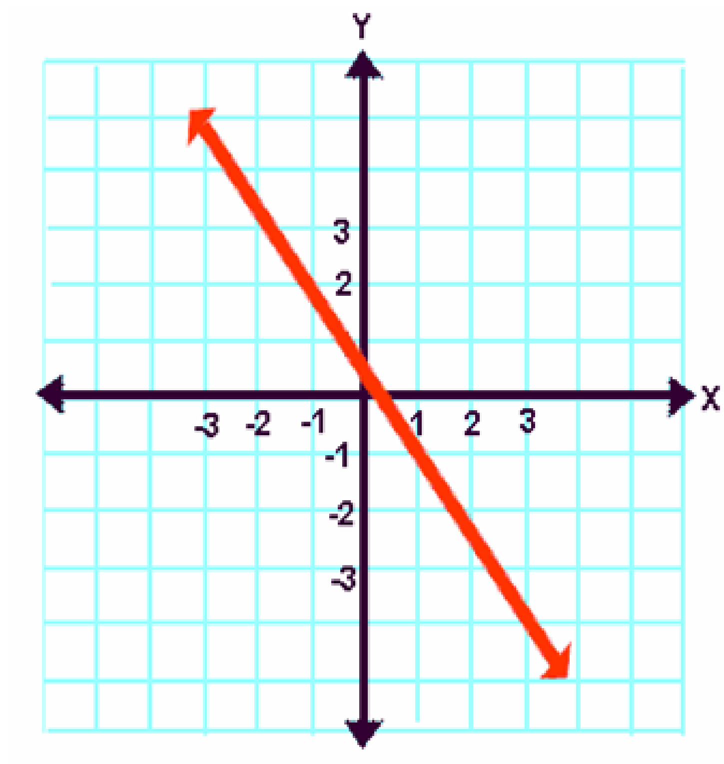
Yes

No

3) The slope of this line is 3/2?

True

False



4) What is the slope of the line $3x + 2y = 12$?

a) 3
b) 3/2
c) -3/2
d) 2

5) Which is the slope of the line through (-2, 3) and (4, -5)?

a) -4/3
b) -3/4
c) 4/3
d) -1/3

6) What is the slope of the line shown in the chart below?

X	1	3	5	7
y	2	5	8	11

a) 1
b) 3/2
c) 3
d) 3/5

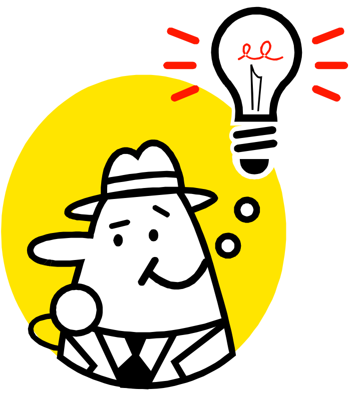
7) Does the line $2y + x = 7$ pass through the point $(1, 3)$?

True

False

8) Which is the equation of a line whose slope is undefined?

a) $x = -5$
b) $y = 7$
c) $x = y$
d) $x + y = 0$



9) Which is the equation of a line that passes through (2, 5) and has slope -3?

a) $y = -3x - 3$

b) $y = -3x + 17$

c) $y = -3x + 11$

d) $y = -3x + 5$

10) Which of these equations represents a line parallel to the line $2x + y = 6$?

a) $y = 2x + 3$

b) $y - 2x = 4$

c) $2x - y = 8$

d) $y = -2x + 1$